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FOURIER–BESSEL FUNCTIONS OF SINGULAR CONTINUOUS MEASURES AND THEIR MANY ASYMPTOTICS*

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Dedicated to Ed Saff on the occasion of his 60th birthday

Abstract. We study the Fourier transform of polynomials in an orthogonal family, taken with respect to the orthogonality measure. Mastering the asymptotic properties of these transforms, that we call Fourier–Bessel functions, in the argument, the order, and in certain combinations of the two is required to solve a number of problems arising in quantum mechanics. We discuss known results, new approaches and open conjectures, hoping to justify our belief that these investigations may involve interesting discoveries, well beyond the quantum mechanical applications.

Key words. singular measures, Fourier transform, orthogonal polynomials, almost periodic Jacobi matrices, Fourier-Bessel functions, quantum intermittency, Julia sets, iterated function systems, generalized dimensions, potential theory

AMS subject classifications. 42C05, 33E20, 28A80, 30E15, 30E20

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