

DISTRIBUTION OF PRIMES AND A WEIGHTED ENERGY PROBLEM*

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Dedicated to Ed Saff on the occasion of his 60th birthday

Abstract. We discuss a recent development connecting the asymptotic distribution of prime numbers with weighted potential theory. These ideas originated with the Gelfond-Schnirelman method (circa 1936), which used polynomials with integer coefficients and small sup norms on $[0, 1]$ to give a Chebyshev-type lower bound in prime number theory. A generalization of this method for polynomials in many variables was later studied by Nair and Chudnovsky, who produced tight bounds for the distribution of primes. Our main result is a lower bound for the integral of Chebyshev's ψ -function, expressed in terms of the weighted capacity for polynomial-type weights. We also solve the corresponding potential theoretic problem, by finding the extremal measure and its support. This new connection leads to some interesting open problems on weighted capacity.

Key words. distribution of prime numbers, polynomials, integer coefficients, weighted transfinite diameter, weighted capacity, potentials

AMS subject classifications. 11N05, 31A15, 11C08

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