

## BERNSTEIN'S WEIGHTED APPROXIMATION ON $\mathbb{R}$ STILL HAS PROBLEMS\*

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*Dedicated to Ed Saff on the occasion of his 60th birthday*

**Abstract.** Let  $W : \mathbb{R} \rightarrow (0, 1]$  be continuous. Bernstein's approximation problem, posed in 1924, dealt with approximation by polynomials in the norm

$$\|f\|_W := \|fW\|_{L^\infty(\mathbb{R})}.$$

The qualitative form of this problem was solved by Achieser, Mergelyan, and Pollard, in the 1950's. Quantitative forms of the problem were actively investigated starting from the 1960's. We survey old and recent aspects of this topic. One recent finding is that there are weights for which the polynomials are dense, but which do not admit a Jackson-Favard inequality. In fact the weight  $W(x) = \exp(-|x|)$  exhibits this peculiarity. Moreover, not all  $L_p$  spaces are the same when degree of approximation is considered. We also pose some open problems.

**Key words.** weighted approximation, polynomial approximation, Jackson-Bernstein theorems

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