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AN INTEGRAL REPRESENTATION OF SOME HYPERGEOMETRIC FUNCTIONS*

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Dedicated to Ed Saff on the occasion of his 60th birthday

Abstract. The Euler integral representation of the ${}_2F_1$ Gauss hypergeometric function is well known and plays a prominent role in the derivation of transformation identities and in the evaluation of ${}_2F_1(a, b; c; 1)$, among other applications. The general ${}_{p+k}F_{q+k}$ hypergeometric function has an integral representation where the integrand involves ${}_pF_q$. We give a simple and direct proof of an Euler integral representation for a special class of ${}_{q+1}F_q$ functions for $q \geq 2$. The values of certain ${}_3F_2$ and ${}_4F_3$ functions at $x = 1$, some of which can be derived using other methods, are deduced from our integral formula.

Key words. 3F2 hypergeometric functions, general hypergeometric functions, integral representation

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