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QUANTUM ALGEBRAS $SU_Q(2)$ AND $SU_Q(1,1)$ ASSOCIATED WITH CERTAIN Q-HAHN POLYNOMIALS: A REVISITED APPROACH*

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Abstract. This contribution deals with the connection of q-Clebsch-Gordan coefficients (q-CGC) of the Wigner-Racah algebra for the quantum groups $SU_q(2)$ and $SU_q(1,1)$ with certain q-Hahn polynomials. A comparative analysis of the properties of these polynomials and $su_q(2)$ and $su_q(1,1)$ Clebsch-Gordan coefficients shows that each relation for q-Hahn polynomials has the corresponding partner among the properties of q-CGC and vice versa. Consequently, special emphasis is given to the calculations carried out in the linear space of polynomials, i.e., to the main characteristics and properties for the new q-Hahn polynomials obtained here by using the Nikiforov-Uvarov approach [A. F. NIKIFOROV, S. K. SUSLOV AND V. B. UVAROV, *Orthogonal Polynomials in Discrete Variables*, Springer-Verlag, Berlin, 1991; A. F. NIKIFOROV AND V. B. UVAROV, *Classical orthogonal polynomials in a discrete variable on non-uniform lattices*, Preprint Inst. Prikl. Mat. M. V. Keldysh Akad. Nauk SSSR (In Russian), 17, Moscow, 1983] on the non-uniform lattice $x(s) = \frac{q^s-1}{q-1}$. These characteristics and properties will be important to extend the q-Hahn polynomials to the multiple case [J. ARVESÚ, *q-Discrete Multiple Orthogonal Polynomials*, in preparation]. On the other hand, the aforementioned lattice allows to recover the linear one $x(s) = s$ as a limiting case, which doesn't happen in other investigated cases [C. CAMPIGOTTO, YU. F. SMIRNOV AND S. G. ENIKEEV, *q-Analogue of the Kravchuk and Meixner orthogonal polynomials*, J. of Comput. and Appl. Math., 57 (1995), pp. 87–97; A. DEL SOL MESA AND YU. F. SMIRNOV, *Clebsch-Gordan and Racah coefficients for $U_q(1,1)$ quantum algebra*, (Discrete series) In Scattering, Reactions, Transitions in Quantum Systems and Symmetry Methods, R.M. Asherova and Yu.F. Smirnov, eds., 1991], for example in $x(s) = q^{2s}$. This fact suggests that the q-analogues presented here (both from the point of view of quantum group theory and special function theory) are *'good'* ones since all characteristics and properties, and consequently, all matrix element relations will converge to the standard ones when q tends to 1.

Key words. Clebsch-Gordan coefficients, discrete orthogonal polynomials (q-discrete orthogonal polynomials), Nikiforov-Uvarov approach, quantum groups and algebras

AMS subject classifications. 81R50, 33D45, 33C80

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