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ITERATIVE SINC-CONVOLUTION METHOD FOR SOLVING RADIOSITY EQUATION IN COMPUTER GRAPHICS*

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Abstract. A new and efficient sinc-convolution algorithm is introduced for the numerical solution of the radiosity equation. This equation has many applications including the production of photorealistic images. The method of sinc-convolution is based on using collocation to replace multi-dimensional convolution-type integrals—such as two dimensional radiosity integral equations—by a system of algebraic equations. The developed algorithm solves for the illumination of a surface or a set of surfaces when both reflectivity and emissivity of those surfaces are known. It separates the radiosity equation's variables to approximate its solution. The separation of variables allows the elimination of the formulation of huge full matrices and therefore reduces required storage, as well as computational complexity, as compared with classical approaches. Also, the highly singular nature of the kernel, which results in great difficulties using classical numerical methods, poses absolutely no difficulties using sinc-convolution. In addition, the new algorithm can be readily adapted for parallel computation for an even faster computational speed. The results show that the developed algorithm clearly reveals the color bleeding phenomenon which is a natural phenomenon not revealed by many other methods. These advantages should make real-time photorealistic image production feasible.

Key words. radiosity, sinc, sinc-convolution, photorealistic, computer graphics

AMS subject classifications. 68U05, 65D18, 65D30

1. Introduction. Global illumination simulation (GIS) is essential for realistic rendering of virtual scenes. In global illumination, we take the geometric definition of a virtual scene and we simulate the propagation of light throughout the scene, modeling its visual and physical effects, such as shadows and reflections [2].

In the past two decades, many computer graphics techniques for simulating the behavior of light interacting with a macroscopic environment have been developed so that the creation of high quality photorealistic images is possible. The radiosity method is one of the favorite methods used in which the exchanges of energy between the objects of the scene are modeled by the so-called "radiosity equation".

The rate at which energy leaves a surface is called its radiosity and it is measured by the rate of energy emitted and reflected from the surface.

Today, much research has been done to achieve a better physically described model and faster methods with less memory cost, s well as more efficient pre-processing (model production processing) and post-processing (rendering) algorithms for solving the radiosity equation. However, there still is a great need for faster and more accurate methods to meet new demands. In this paper, a new and efficient algorithm based on the *sinc-convolution* method is introduced to meet most of these requirements.

Since the 2-D integrals of the radiosity equation are integral equations of convolution type, the method of *sinc-convolution* can easily be adapted for its accurate approximation. The *sinc-convolution* method overcomes the formation of huge full matrices in the solution of multidimensional integral equations which arise in solving the radiosity equation using classical approaches. Thus, the method reduces the computational complexity by a large amount. In addition, the highly singular nature of the kernel of the radiosity integral equation, which causes great difficulty using classical numerical methods, poses absolutely no difficulty using the sinc-convolution method. Moreover, the new algorithm can readily be adapted for

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parallel computation for even faster computational speed. These advantages make it possible to achieve real time high quality photorealistic digital images.

2. Radiosity equation. The radiosity equation is a mathematical model for the brightness of a collection of surfaces when their reflectivity and emissivity are given. This equation is classified as a linear system of two-dimensional singular linear Fredholm equations of the second kind and is given by [1]

$$B(P) - \frac{\rho(P)}{\pi}\int_s B(Q)g(P,Q)V(P,Q)dS_Q = E(P), \qquad P \in S,$$

where $B(P)$ is the brightness or radiosity at point $P, E(P)$ is the function of emissivity at $P \in S$, the function $\rho(P)$ gives the reflectivity at $P \in S$ where $0 \le \rho(P) < 1$, and the function $V(P,Q)$ is a "line of sight" function. If the points P and Q could "see each other" along a straight line segment which does not intersect surface S at any other point, then $V(P,Q) = 1$; otherwise, $V(P,Q) = 0$. It is assumed that the surface is a Lambertian diffuse reflector surface, which means its reflectivity is assumed to be uniform in all directions [1, 5]. The kernel function, $g(P,Q)$, is given by [1]

$$g(P,Q) = \frac{\cos\theta_P\cos\theta_Q}{|P-Q|^2} = \frac{[(Q-P).n_p][(P-Q).n_Q]}{|P-Q|^4}.$$

In the kernel function, n_P is the inner unit normal to surface S at point P and θ_P is the angle between n_P and line PQ. n_Q and θ_Q are defined analogously. For example, for the surfaces of an empty room with $M - by - N - by - L$ dimensions,

$$\begin{aligned} &S_{xz} = \{(x,0,z)|0 \le x \le M, 0 \le z \le L\}, \\ \text{and} \quad &S_{xy} = \{(x,y,0)|0 \le x \le M, 0 \le y \le N\}, \end{aligned}$$

the kernel function g would become

$$g(P,Q) = \begin{cases} \dfrac{bz}{[(x-a)^2+b^2+z^2]^2}, & P \in S_{xy}, \quad Q \in S_{xz}, \\ \dfrac{yc}{[(x-a)^2+y^2+c^2]^2}, & P \in S_{xz}, \quad Q \in S_{xy}, \\ 0, & \text{otherwise}, \end{cases}$$

where $P = (a,b,c)$ and $Q = (x,y,z)$. To solve the radiosity equation, it is easier to write it in the parametric form. In the two dimensional parametric form of the radiosity equation, the key variables, including reflectivity, emissivity, and radiosity, would be two-variable functions for the i^{th} surface as receiver and j^{th} surface as transmitter of light energy as follows:

$$B_i(s,t) - \rho_i(s,t)\sum_j \int\int B_j(u,v)G_{i-j}(s,t,u,v)V(s,t,u,v)dA_j(u,v)dudv,$$

where $B_i(s,t)$ is the radiosity, $E_i(s,t)$ is the emissivity, and $\rho_i(s,t)$ is the Lambertian reflection at point (s,t) on the i^{th} surface; and $dA_j(u,v)$ is the differential area at point (u,v) of the j^{th} surface. In addition, $G_{i-j}(s,t,u,v)$ is the form factor and $V(s,t,u,v)$ is the "line of sight" both from point (s,t) on the i^{th} surface to the point (u,v) on the j^{th} surface.

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We combine the functions of form factors, differential area, and the "line of sight" and in $k_{ij}(s,t,u,v)$, the kernel function. Then, the radiosity equation becomes:

$$B_i(s,t) - \rho_i(s,t) \sum_j \int \int k_{ij}(s,t,u,v) B_j(u,v) du dv. \tag{2.1}$$

In the next section, a *sinc-convolution* approximation algorithm is developed to solve the radiosity equation defined in (2.1).

3. *Sinc-convolution* approximation for solving radiosity equation. Here, we briefly describe an iterative *sinc-convolution* method and we apply it to solve the radiosity equation. The detailed description of the method is available in many publications including [3, 7, 9, 12, 13]. To solve the radiosity equation, we assume one source of light in an empty room such that any surface S in the room is assumed to be un-occluded. We, further, assume that the room's length is M, its breadth is N, and its height is L. We use the *sinc-convolution* method to replace the integral equation in (2.1) by a system of algebraic equations in all dimensions. In effect, we develop a quadrature formula to evaluate the integral equation using a separation of variables technique.

Let's assume that there are $L+1$ number of surfaces, $\mathbf{S}_k$, for $k = 0, \cdots, L$, on the z axis inside the room such that there are $(\mathsf{M}+1) \times (\mathsf{N}+1)$ samples of sinc points, selected on each of the surfaces, along the x and y axis, respectively, where we would like to calculate the illumination using the the two dimensional radiosity equation.

Now, in each surface, we set $m_i = M_i + N_i + 1$ for $i = 1, 2$ and define the sinc points by $z_k^i = \phi_i^{-1}(kh_i)$ where ϕ_i^{-1} denotes the inverse function of the mapping functions ϕ_i. The selection of the mapping functions, ϕ_i, depends upon specific regions. An extensive analysis of selecting a proper mapping function for any specific region is available in [9, 10, 13].

Based on the regions in our problem, the best mapping function for the x transformation would be $\phi(x) = \ln\left(\frac{x}{M-x}\right)$ which determines the sinc points $x_i = \frac{Me^{ih_1}}{e^{ih_1}+1}, i = -M, \cdots, M$ [10]. The *sinc-convolution* method requires the derivative of the mapping function [9, 10, 13]. This is known to be $\phi'(x) = \frac{M}{x(M-x)}$[9, 10]. The same mapping function may also be used for other axis transformations.

Now, let z denote the set of all integers and let σ_k be defined by

$$\sigma_k = \int_0^k \frac{\sin(\pi z)}{\pi z} dz, \quad k \in Z.$$

Also, let $I_{m_i}^{(-1)}$ be a square matrix of order m_i having $\delta_{j-l}^{(-1)}$ as its element where

$$\delta_k^{(-1)} = \sigma_k + \frac{1}{2}.$$

We determine matrices A_i, X_i, S_i and X_i^{-1} and such that [8, 9, 11, 13]:

$$A_1 = h_1 I_{m_1}^{(-1)} D_1 \left(\frac{1}{\phi'(x)}\right) = X_1 S_1 X_1^{-1} \tag{3.1}$$

$$A_{2\_1} = h_2 I_{m_2}^{(-1)} D_2 \left(\frac{1}{\phi'(y)}\right) = X_{2\_1} S_{2\_1} X_{2\_1}^{-1} \tag{3.2}$$

$$A_{2\_2} = h_2 \left[I_{m_2}^{(-1)}\right]^T D_2 \left(\frac{1}{\phi'(y)}\right) = X_{2\_2} S_{2\_2} X_{2\_2}^{-1} \tag{3.3}$$

$$A_3 = h_3 \left[I_{m_3}^{(-1)}\right]^T D_3 \left(\frac{1}{\phi'(z)}\right) = X_3 S_3 X_3^{-1} \tag{3.4}$$

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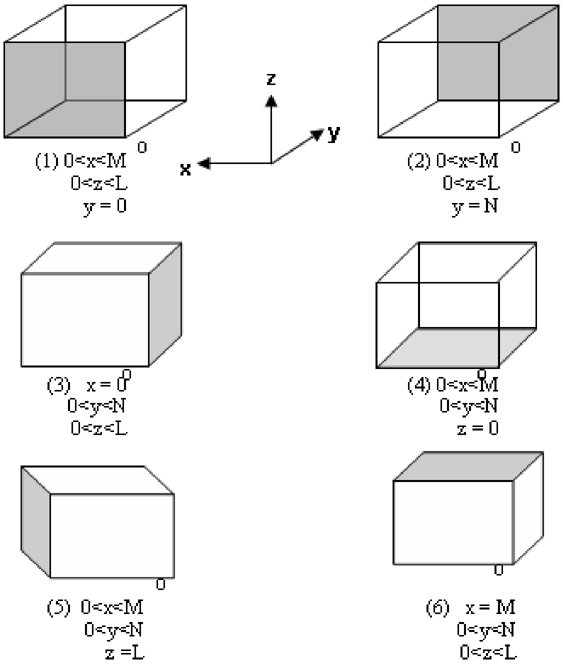

FIG. 3.1. *The geometry of the scene is a $M - by - N - by - L$ room with origin at O.*

where $X_i, i = 1, 2\_1, 2\_2, 3$ are matrices of eigenvectors and

$$D_i(u) = diag\left[u\left(z_{-M_i}^{(i)}\right), \cdots, u\left(z_{N_i}^{(i)}\right)\right],$$

where z_j^i shows sinc points for all three dimensions and the diagonal matrix

$$S_i = diag\left[s_{-M_i}^i, \cdots, s_{N_i}^i\right],$$

with s_j^i as the eigenvalues [3].

As we mentioned before, an empty room is considered as the geometry of our scene where there is no occluded surface. We further assume that the room's walls have different colors. The geometry of the scene is shown in Figure 3.1.

Since the considered scene is an empty room, there is no object to block the "lines of sight" and so the value for V in the kernel function is always equal to 1. With V=1, the kernels of all surfaces are derived and are tabulated in the Table 3.1.

Following the *sinc-convolution* algorithm from [7, 8, 11, 12, 13], a special version of "Laplace transform" of the kernel is required.

3.1. Laplace transform. The sinc-convolution algorithm requires a special form of "Laplace transform" of kernel of the integral, $g(P, Q)$, denoted in quotes, where its Laplace variables are inverted. It is defined by [3]

$$G(s, v) = \int_0^\infty \int_0^\infty g(x, y) e^{-\left(\frac{x}{s} + \frac{y}{v}\right)} dx dy.$$

For the kernels given in Table 3.1, we need to compute the "Laplace transform" of the following integrals:

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$$\int_0^\infty \int_0^\infty \frac{z}{(x-a)^2+(y-b)^2+(z-c)^2} e^{\frac{-x}{s}} e^{\frac{-z}{w}} dxdz,$$
$$\int_0^\infty \int_0^\infty \frac{z}{(x-a)^2+(y-b)^2+(z-c)^2} e^{\frac{-y}{v}} e^{\frac{-z}{w}} dydz,$$
$$\int_0^\infty \int_0^\infty \frac{y}{(x-a)^2+(y-b)^2+(z-c)^2} e^{\frac{-y}{v}} e^{\frac{-z}{w}} dydz,$$
$$\int_0^\infty \int_0^\infty \frac{y}{(x-a)^2+(y-b)^2+(z-c)^2} e^{\frac{-x}{s}} e^{\frac{-y}{v}} dxdy,$$
$$\int_0^\infty \int_0^\infty \frac{x}{(x-a)^2+(y-b)^2+(z-c)^2} e^{\frac{-x}{s}} e^{\frac{-z}{w}} dxdz,$$
$$\int_0^\infty \int_0^\infty \frac{x}{(x-a)^2+(y-b)^2+(z-c)^2} e^{\frac{-x}{s}} e^{\frac{-y}{v}} dxdy,$$
$$\int_0^\infty \int_0^\infty \frac{1}{(x-a)^2+(y-b)^2+(z-c)^2} e^{\frac{-x}{s}} e^{\frac{-y}{v}} dxdy,$$
$$\int_0^\infty \int_0^\infty \frac{1}{(x-a)^2+(y-b)^2+(z-c)^2} e^{\frac{-x}{s}} e^{\frac{-z}{w}} dxdz,$$
$$\int_0^\infty \int_0^\infty \frac{1}{(x-a)^2+(y-b)^2+(z-c)^2} e^{\frac{-y}{v}} e^{\frac{-z}{w}} dydz. \tag{3.5}$$

The first dimension of the "Laplace transform" integrals of the radiosity equation is computed analytically. However, the second dimension could not be solved analytically and therefore we solve it using *Gauss-Legendre* numerical method. Here, we only demonstrate the analytical solution procedure for the following two integrals. We extend the procedure to the other involved integrals by adaptation [3].

$$\int_0^\infty \frac{1}{(x-a)^2+(y-b)^2+(z-c)^2} e^{\frac{-x}{s}} dx, \tag{3.6}$$

$$\int_0^\infty \frac{x}{(x-a)^2+(y-b)^2+(z-c)^2} e^{\frac{-x}{s}} dx. \tag{3.7}$$

We assume $\beta = \sqrt{(y-b)^2+(z-c)^2}$, and find the kernel for the above intergrals, $f(x)$, as

$$f(x) = \frac{x}{\beta^2+(x-a)^2}.$$

Now, if we assume the "Laplace transform" of function $f(t)$ as $F(s)$, then

$$L\{f'(t)\} = sF(s) - f(0).$$

Solving the above equation, we have [4, 6]

$$L\{f'(t)\} = (a^2+\beta^2) \int_0^\infty \frac{1}{[\beta^2+(x-a)^2]^2} e^{\frac{-x}{s}} dx - \int_0^\infty \frac{x^2}{[\beta^2+(x-a)^2]^2} e^{\frac{-x}{s}} dx.$$

With the change of variable as $x - a = X$ for the second integral and performing some simplification, we arrive at the expression

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TABLE 3.1
Kernel functions for the surfaces of an empty room.

$$k_{11} = 0 \qquad k_{12} = \frac{N^2}{\left[(x-a)^2 + N^2 + (z-c)^2\right]^2}$$

$$k_{13} = \frac{ay}{\left[(a^2 + N^2 + (z-c)^2\right]^2} \qquad k_{14} = \frac{yc}{\left[(x-a)^2 + y^2 + c^2\right]^2}$$

$$k_{15} = \frac{y(L-c)}{\left[(x-a)^2 + y^2 + (L-c)^2\right]^2} \qquad k_{16} = \frac{y(M-a)}{\left[(M-a)^2 + y^2 + (z-c)^2\right]^2}$$

$$k_{21} = \frac{N^2}{\left[(x-a)^2 + N^2 + (L-c)^2\right]^2} \qquad k_{22} = 0$$

$$k_{23} = \frac{a(N-y)}{\left[a^2 + (y-N)^2 + (z-c)^2\right]^2} \qquad k_{24} = \frac{c(N-y)}{\left[(x-a)^2 + (y-N)^2 + c^2\right]^2}$$

$$k_{25} = \frac{(L-c)(N-y)}{\left[(x-a)^2 + (y-N)^2 + (L-c)^2\right]^2} \qquad k_{26} = \frac{(M-a)(N-y)}{\left[(M-a)^2 + (y-N)^2 + (z-c)^2\right]^2}$$

$$k_{31} = \frac{xb}{\left[x^2 + b^2 + (z-c)^2\right]^2} \qquad k_{32} = \frac{x(N-b)}{\left[x^2 + (N-b)^2 + (z-c)^2\right]^2}$$

$$k_{33} = 0 \qquad k_{34} = \frac{xc}{\left[x^2 + (y-b)^2 + c^2\right]^2}$$

$$k_{35} = \frac{x(L-c)}{\left[x^2 + (y-b)^2 + (L-c)^2\right]^2} \qquad k_{36} = \frac{M^2}{\left[M^2 + (y-b)^2 + (z-c)^2\right]^2}$$

$$k_{41} = \frac{zb}{\left[(x-a)^2 + b^2 + z^2\right]^2} \qquad k_{42} = \frac{z(N-b)}{\left[(x-a)^2 + (N-b)^2 + z^2\right]^2}$$

$$k_{43} = \frac{az}{\left[a^2 + (y-b)^2 + z^2\right]^2} \qquad k_{44} = 0$$

$$k_{45} = \frac{L^2}{\left[(x-a)^2 + (y-b)^2 + L^2\right]^2} \qquad k_{46} = \frac{z(M-a)}{\left[(M-a)^2 + (y-b)^2 + z^2\right]^2}$$

$$k_{51} = \frac{b(L-z)}{\left[(x-a)^2 + b^2 + (z-L)^2\right]^2} \qquad k_{52} = \frac{(N-b)(L-z)}{\left[(x-a)^2 + (N-b)^2 + (z-L)^2\right]^2}$$

$$k_{53} = \frac{a(L-z)}{\left[a^2 + (y-b)^2 + (z-L)^2\right]^2} \qquad k_{54} = \frac{L^2}{\left[(x-a)^2 + (y-b)^2 + (z-L)^2\right]^2}$$

$$k_{55} = 0 \qquad k_{56} = \frac{(M-a)(L-z)}{\left[(M-a)^2 + (y-b)^2 + (z-L)^2\right]^2}$$

$$k_{61} = \frac{b(M-x)}{\left[(x-M)^2 + b^2 + (z-c)^2\right]^2} \qquad k_{62} = \frac{(N-b)(M-x)}{\left[(x-M)^2 + (N-b)^2 + (z-c)^2\right]^2}$$

$$k_{63} = \frac{M^2}{\left[M^2 + (y-b)^2 + (z-c)^2\right]^2} \qquad k_{64} = \frac{c(M-x)}{\left[(x-M)^2 + (y-b)^2 + c^2\right]^2}$$

$$k_{65} = \frac{(L-c)(M-x)}{\left[(x-M)^2 + (y-b)^2 + (L-c)^2\right]^2} \qquad k_{66} = 0.$$

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$$\int_0^\infty \frac{1}{[\beta^2+(x-a)^2]^2} e^{\frac{-x}{s}} dx = \frac{1}{\beta^2}\left[e^{\frac{-a}{s}} \int_{-a}^\infty \frac{x^2 e^{\frac{-x}{s}}}{[\beta^2+x^2]^2} dx \right.$$
$$\left. +2ae^{\frac{-a}{s}} \int_{-a}^\infty \frac{x e^{\frac{-x}{s}}}{[\beta^2+x^2]^2} dx + sF(s) \right].$$

The values of the right integrals are known and they are documented in [6]. Now with proper replacements, the integral (3.6) is obtained as [3]

$$\int_0^\infty \frac{1}{(x-a)^2+(y-b)^2+(z-c)^2} e^{\frac{-x}{s}} dx = \frac{a}{2\beta^2(\beta^2+a^2)} + \frac{1}{4s\beta^2} \times$$
$$\times \left[E_i\left(\frac{-a}{s}+i\frac{\beta}{s}\right) exp\left(\frac{-a}{s}+i\frac{\beta}{s}\right) + E_i\left(\frac{-a}{s}-i\frac{\beta}{s}\right) exp\left(\frac{-a}{s}-i\frac{\beta}{s}\right)\right]$$
$$-i\frac{1}{4\beta^3}\left[-E_i\left(\frac{-a}{s}+i\frac{\beta}{s}\right) exp\left(\frac{-a}{s}+i\frac{\beta}{s}\right) + E_i\left(\frac{-a}{s}-i\frac{\beta}{s}\right) exp\left(\frac{-a}{s}-i\frac{\beta}{s}\right)\right],$$

where

$$Ei(x) = \int_x^\infty \frac{e^{-t}t}{d} t.$$

With a similar procedure, we also obtain a solution for (3.7) as [3, 6]

$$\int_0^\infty \frac{x}{(x-a)^2+(y-b)^2+(z-c)^2} e^{\frac{-x}{s}} dx = \frac{1}{2\beta^2} + \frac{a}{4s\beta^2} \times$$
$$\times \left[E_i\left(\frac{-a}{s}+i\frac{\beta}{s}\right) exp\left(\frac{-a}{s}+i\frac{\beta}{s}\right) + E_i\left(\frac{-a}{s}-i\frac{\beta}{s}\right) exp\left(\frac{-a}{s}-i\frac{\beta}{s}\right)\right] +$$
$$+i\left(\frac{1}{4s\beta} - \frac{a}{4\beta^3}\right)\left[-E_i\left(\frac{-a}{s}+i\frac{\beta}{s}\right) exp\left(\frac{-a}{s}+i\frac{\beta}{s}\right) + \right.$$
$$\left. +E_i\left(\frac{-a}{s}-i\frac{\beta}{s}\right) exp\left(\frac{-a}{s}-i\frac{\beta}{s}\right)\right].$$

The first integration of the other 2-D integrals of (3.5) are solved in a similar manner. We could not solve the second integral analytically. Therefore, we solved it numerically using *Gauss-Legendre* numerical method. In the next section, we illustrate the *sinc-convolution* algorithm using the method of separation of variables.

3.2. Separation of variables of 2-D integrals. Here, we enlist the method of separation of variables for solving the two-dimensional convolution-type integrals defined in (3.8) below, used in the radiosity integral equations of (2.1), and rewritten in the form

$$B(P) = \frac{\rho(P)}{\pi} p(a,b) = E(P), \quad P \in S,$$

where

$$p(a,b) = \int_0^N \int_0^M C(x,y) g(x-a, y-b) dx dy, \tag{3.8}$$

and $C(.,.) = B(.)V(.,.)$. We omit the derivation of the algorithm and refer the readers to the references [9, 13] for in depth analysis of the method as well as its derivation. The method is summarized in Table 3.2. The separation of variables of all the 2-D integrals in the radiosity equation is done analogously [3].

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TABLE 3.2
Algorithm for approximate solution of $p(a,b)$ at the set of sinc points.

1. Form the array of sinc point $z_j^{(k)}$ and $\frac{d}{dx}\phi^{(k)}(x)$ at $x = z_j^{(k)}$ for $k = 1, 2$ and $j = -M_k, \cdots, M_k$, and then form the array $u_{i,j} = [C(z_i^{(1)}, z_j^{(2)})]$.

2. Determine A_i, X_i, X_i^{-1} and $S_i, i = 1, 2\_1, 2\_2, 3$ as defined in (3.1) through (3.4).

3. Successively compute:

$$b_{\cdot j}^{+} = X_1^{-1} u_{\cdot j}$$

$$t_{i,\cdot}^{+} = X_2^{-1} b_{i,\cdot}$$

4. Form the products:

$$t_{i,j}^{-} = G_{xy}(s_i^{(1)}, s_j^{(2)}) t_{i,j}^{+}$$

where $G_{xy}(s_i^{(1)}, s_j^{(2)})$ is the "Laplace transform" of $g(x,y)$ defined in (3.1).

5. Successively form the arrays:

$$b_{i,\cdot}^{-} = X_1 t_{i,\cdot}^{-}$$

$$p_{\cdot,j} = X_1 b_{\cdot,j}^{-}$$

Note: $P_{i,j}$ are the approximations of $p(a,b)$ at the sinc points.

3.3. Sinc basis interpolation. Now, we can use the obtained values of radiosity at the sinc points to approximate the radiosity values in all points of the scene via the use of sinc basis interpolation. First, we define function B below [7, 8, 9] as

$$B(x,y) \approx \sum_{i=-M_1}^{N_1} \sum_{j=-M_2}^{N_2} B(x_i, y_j) w_i^{(1)} w_j^{(2)}(y),$$

where

$$w_n^{(m)} = \frac{\sin\left[\frac{\pi}{h_m}(\phi_m - nh_m)\right]}{\frac{\pi}{h_m}(\phi_m - nh_m)} = \operatorname{sinc}\left(\frac{\phi_m - nh_m}{h_m}\right).$$

With $m = 1, 2$ and $n = -M_m + 1, \cdots, N_m - 1$, we have [7, 9, 11, 13]

$$w_{-M}^{(m)} = \frac{1}{1+\rho_m} - \sum_{n=-M+1}^{N} \left[\frac{1}{1+e^{nh_m}} w_n^{(m)}\right],$$

$$w_N^{(m)} = \frac{\rho_m}{1+\rho_m} - \sum_{n=-M}^{N-1} \left[\frac{e^{nh_m}}{1+e^{nh_m}} w_n^{(m)}\right],$$

where $\rho_m = e^{\phi_m}$.

4. Approximation results. A computer code has been developed by the authors for implementing the iterative *sinc-convolution* method for solving the radiosity integral equation problem in an empty room with one source of light.

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For the purpose of comparison between photorealistic and non-photorealistic images, first, a non-photorealistic image of the room is presented and then two samples of images from the images created by our program is represented [3].

Figure 4.1 shows a non photorealistic image of a given room with one light source and Figure 4.2 shows an image that is created by our program having seven surfaces with 7-by-7 sinc points on each one of them, and Figure 4.3 shows the results assuming nine surfaces with 9-by-9 sinc points on each of the surfaces.

The results show that our algorithm works well for solving the radiosity problem despite a great decrease of computational complexity. Naturally, increasing the number of sinc points causes more realistic and accurate images. The issue of computational complexity is addressed in the following subsection.

4.1. Computational complexity. The major advantage of the *sinc-convolution* method over existing methods is a remarkable reduction in computational complexity. In this section, we compare the computational complexity of the new method with that of the traditional finite-difference methods in terms of the work required by a computer to achieve desired precision. To this end, we assume an equal number of discrete points for all three dimensions, say M. Then, the *sinc-convolution* algorithm for solving our radiosity equation would require $3000 \times M^3$ complex operations counting 5 matrix multiplications for approximating each of the 30 integrals in each of the (x, y) or (x, z) or (y, z) directions and assuming 10 iterations to achieve convergence, $(3000 = 5 \times 30 \times 2 \times 10)$.

In addition, the method guarantees an accuracy to within a uniform error of ε , with [7]

$$\varepsilon \approx e^{-cM^{\frac{1}{2}}}.$$

Thus, by eliminating M, we get an expression for the complexity $W_{SINC}(\varepsilon)$, i.e., the work required to achieve an accuracy of ε by the *sinc-convolution* method [7],

$$W_{SINC}(\varepsilon) = \frac{3000}{c^6}\left(\log\frac{1}{\varepsilon}\right)^6,$$

where c is a constant and usually $c \approx \pi$.

For finite-difference methods, let us assume the space step size $h = \frac{2c''}{M^{1/2}}$. The method's error is known to be [4]

$$\varepsilon = O\left(3h^2\right).$$

Using the *sinc-convolution* method, there is no need for any extra computations for singular points, and so we did not consider that in the above relations. In addition, we did not consider the complexity of solving for the "Laplace transform" of the radiosity equation's kernel using *Gauss-Legendre* numerical method. This is justified since the kernel always remains the same, and therefore, one doesn't need to compute the "Laplace transform" of the kernel for a specific number of sinc points more than once. The result could be stored for solving further radiosity photorealistic problems with the same number of sinc points.

The required number of complex operations for the finite-difference method, thus, is of the order of $O\left(c_k M^2\right)$ for some constant $c_k \gg 1$. We selected $c_k = 30$ [7]. Thus, the minimum work (i.e. complexity) required in terms of $\varepsilon - accuracy$ for solving the radiosity equation using a finite-difference method is

$$W_{FD}(\varepsilon) \geq \frac{c_k \times (c'')^4}{\varepsilon^2}.$$

A plot of complexity vs. $\varepsilon-$accuracy for both *sinc-convolution* and *finite-difference* methods is given in Figure 4.4.

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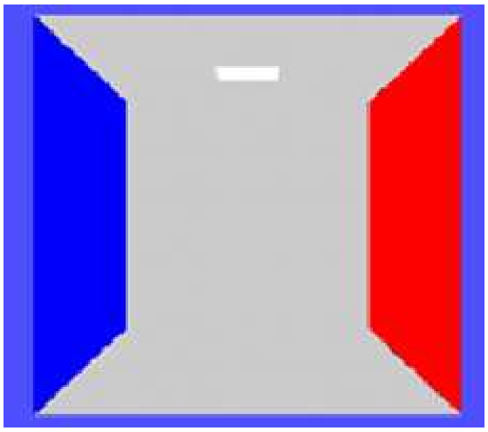

FIG. 4.1. *Non-photorealistic image of a room.*

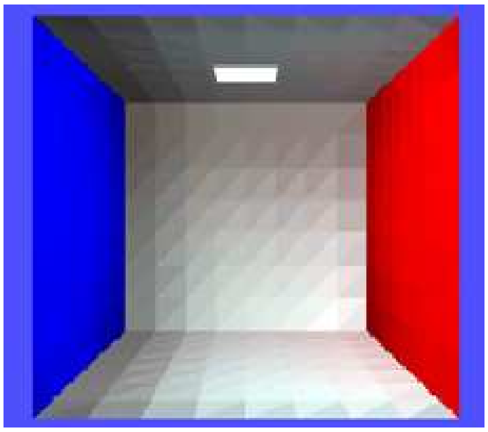

FIG. 4.2. *Photorealistic image of the room with 343 sinc points.*

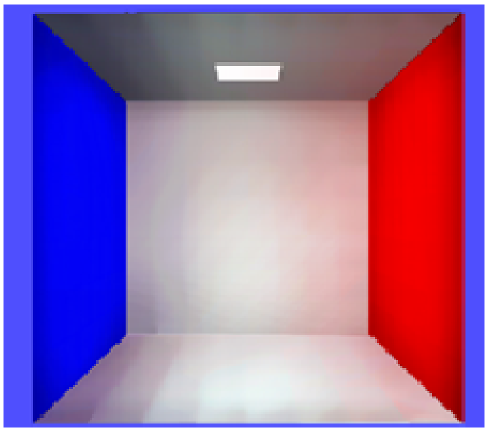

FIG. 4.3. *Photorealistic image of the room with 729 sinc points.*

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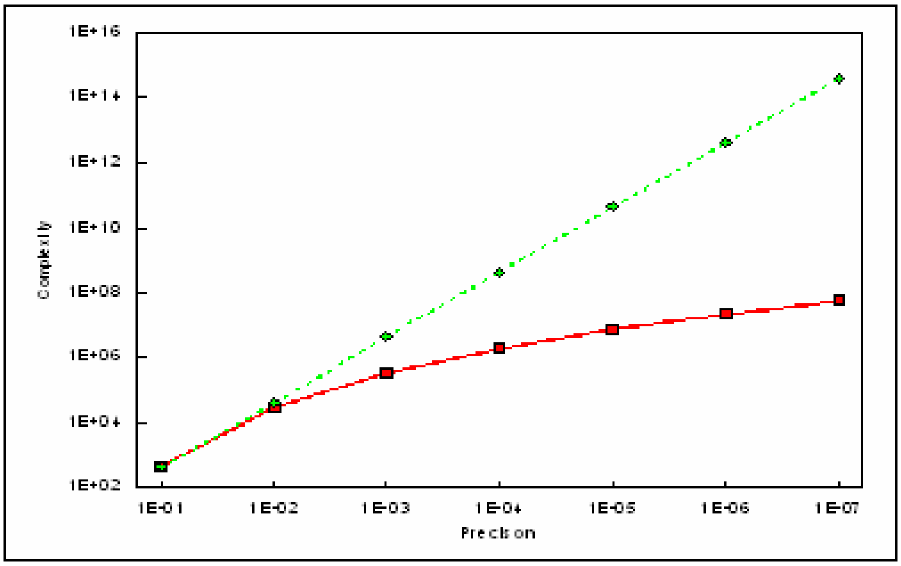

FIG. 4.4. *The complexity vs. precision, ε, plot for both* sinc-convolution *and* finite-difference *methods.*

5. Concluding remarks. In this paper, a new and efficient integral equation approach called *sinc-convolution* is used to solve the two-dimensional radiosity integral equation problem. The *sinc-convolution* algorithm overcomes the formation of huge full matrices and therefore circumvents the expensive storage and computational complexity difficulties that are encountered in solving the large matrix problems that are required of the other integral-equation approaches.

In effect, the *sinc-convolution* method is a quadrature formula which enables the simultaneous evaluation of the 2-D integrals of convolution type at the given sinc points.

The computational results suggest that using a larger number of the sinc points in the x and y and z directions will improve the performance of the method. This is expected, since a larger number of sinc points increases the resolution at the cost of higher computational complexity. In addition, the new algorithm could readily be adapted to parallel computations. The parallel format results in even faster computational speed. As a practical matter, these advantages should make real-time computations feasible.

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