

TIME-DISCRETIZATION OF A DEGENERATE REACTION-DIFFUSION EQUATION ARISING IN BIOFILM MODELING*

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Abstract. A numerical method for a reaction-diffusion equation arising in biofilm modelling is presented. The equation shows two non-standard effects in the spatial operator, degeneracy like the porous medium equation and a singularity as an *a priori* known upper bound is approached. The equation is transformed and formulated in terms of a new dependent variable. This transformation is chosen such that the resulting spatial operator is the Laplace operator and that the non-linear effects appear now in the time-derivative. The numerical method for the new equation follows Rothe's approach: while a standard discretization for the spatial domain is used, a fully-implicit time-discretization scheme is developed that takes the special properties of the equation into account. This paper presents the formulation of this time-discretization scheme as well as its analysis.

Key words. degenerate parabolic equations, nonlinear diffusion, numerical solution, biofilm model

AMS subject classifications. 35K65, 65M12, 92B05

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