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Contents

time-discretization scheme is developed that takes the special properties of the equation into account. This paper presents the formulation of this time-discretization scheme as well as its analysis.

Key Words.
degenerate parabolic equations, nonlinear diffusion, numerical solution, biofilm model

AMS(MOS) Subject Classifications.
35K65, 65M12, 92B05

38 Isotropic and anisotropic a posteriori error estimation of the mixed finite element method for second order operators in divergence form. *Serge Nicaise and Emmanuel Creusé*.

Abstract.
This paper presents an a posteriori residual error estimator for the mixed FEM of second order operators using isotropic or anisotropic meshes in $\mathbb{R}^d$, $d = 2$ or 3. The reliability and efficiency of our estimator is established without any regularity assumptions on the solution of our problem.

Key Words.
error estimator, stretched elements, mixed FEM, anisotropic solution

AMS(MOS) Subject Classifications.
65N15, 65N30

63 Toward the Sinc-Galerkin method for the Poisson problem in one type of curvilinear coordinate domain. *Toshihiro Yamamoto*.

Abstract.
This paper introduces the Sinc-Galerkin method for the Poisson problem in one type of curvilinear coordinate domain and shows an example of the numerical results. The method proposed in this paper transforms the domain of the Poisson problem designated by the curvilinear coordinates into a square domain. In this process, Poisson's equation is transformed into a more general two-variable second-order linear partial differential equation. Therefore, this paper also shows a unified solution for general two-variable second-order linear partial differential equations. The derived matrix equation is represented by a simple matrix equation by the use of the Kronecker product. However, the implementation for real applications requires a more efficient calculation of the matrix equation.

Key Words.
Sinc-Galerkin method, sinc methods, Poisson problem, differential equations

AMS(MOS) Subject Classifications.
65N99

76 Quasi-Newton preconditioners for the inexact Newton method. *L. Bergamaschi, R. Bru, A. Martínez, and M. Putti*.

Abstract.
In this paper preconditioners for solving the linear systems of the Newton method in each nonlinear iteration are studied. In particular, we define a sequence of preconditioners built by means of Broyden-type rank-one updates. Optimality conditions

are derived which guarantee that the preconditioned matrices are not far from the identity in a matrix norm. Some notes on the implementation of the corresponding inexact Newton method are given and some numerical results on two model problems illustrate the application of the proposed preconditioners.

Key Words.
Quasi-Newton method, Krylov iterations, updating preconditioners, inexact Newton method

AMS(MOS) Subject Classifications.
65F10, 65H10, 15A12

88 Solution of singular elliptic PDEs on a union of rectangles using sinc methods. *Michael H. Hohn*.

Abstract.
The numerical solution of problems with singularities presents special difficulties for most methods. Adjustments to standard methods are typically made for only a special type of singularity, usually known a priori. The family of sinc numerical methods is natually suited for general singular problems. Here, the methods are extended and applied to two-dimensional, elliptic first-order systems of mixed boundary value problems with singularities of the form x^{α}.

Key Words.
elliptic, PDE, sinc methods, singular problems, BVP

AMS(MOS) Subject Classifications.
65N35, 65N12, 74S25, 74G15, 74G70

105 Scalable algebraic multigrid on 3500 processors. *Wayne Joubert and Jane Cullum*.

Abstract.
A parallel algebraic multigrid linear solver method is presented which is scalable to thousands of processors on significant classes of two- and three-dimensional problems. The algorithm is entirely algebraic and does not require prior information on the physical problem. Scalability is achieved through the use of an innovative parallel coarsening technique in addition to aggressive coarsening and multipass interpolation techniques. Details of this algorithm are presented together with numerical results on up to several thousand processors.

Key Words.
algebraic multigrid, AMG, parallel computing, simulations, scalable, linear solvers, parallel coarsening

AMS(MOS) Subject Classifications.
65F10

129 The Sinc-Galerkin method for solving singularly-perturbed reaction-diffusion problem. *Mohamed El-Gamel*.

Abstract.
One of the new techniques used in solving boundary-value problems involving partial differential equations is the Sinc-Galerkin method. In this paper we solve the singularly-perturbed reaction-diffusion problem using the Sinc-Galerkin method.

The scheme is tested on four problems and a comparison with finite element methods and the method of reduction of order is made. It is show that the Sinc-Galerkin method yields better results.

Key Words.
sinc function, Sinc-Galerkin, singularly perturbed, reaction-diffusion, numerical solutions.

AMS(MOS) Subject Classifications.
65N30,65T60, 35K05, 42C40

141 On the reduction of a Hamiltonian matrix to Hamiltonian Schur form. *David S. Watkins*.

Abstract.
Recently Chu, Liu, and Mehrmann developed an $O(n^3)$ structure preserving method for computing the Hamiltonian real Schur form of a Hamiltonian matrix. This paper outlines an alternative derivation of the method and an alternative explanation of why the method works. Our approach places emphasis eigenvalue swapping and relies less on matrix manipulations.

Key Words.
Hamiltonian matrix, skew-Hamiltonian matrix, stable invariant subspace, real Schur form

AMS(MOS) Subject Classifications.
65F15, 15A18, 93B40

158 On fast factorization pivoting methods for sparse symmetric indefinite systems. *Olaf Schenk and Klaus Gärtner*.

Abstract.

This paper discusses new pivoting factorization methods for solving sparse symmetric indefinite systems. As opposed to many existing pivoting methods, our Supernode–Bunch–Kaufman (SBK) pivoting method dynamically selects 1×1 and 2×2 pivots and may be supplemented by pivot perturbation techniques. We demonstrate the effectiveness and the numerical accuracy of this algorithm and also show that a high performance implementation is feasible. We will also show that symmetric maximum-weighted matching strategies add an additional level of reliability to SBK. These techniques can be seen as a complement to the alternative idea of using more complete pivoting techniques during the numerical factorization. Numerical experiments validate these conclusions.

Key Words.
direct solver, pivoting, sparse matrices, graph algorithms, symmetric indefinite matrix, interior point optimization

AMS(MOS) Subject Classifications.
65F05, 65F50, 05C85

180 Parameter-uniform fitted mesh method for singularly perturbed delay differential equations with layer behavior. *M. K. Kadalbajoo and K. K. Sharma*.

Abstract.
Boundary value problems for singularly perturbed differential difference equations containing delay with layer behavior are considered. There are a number of realistic models in the literature where one encounters BVPs for singularly perturbed differential difference equations with small delay, such as in variational problems in control theory and first exit time problems in modeling of activation of neurons. In some recent papers, the terms negative shift for 'delay' and positive shift for 'advance' are used. In this paper, a numerical method based on the fitted mesh approach to approximate the solution of these types of boundary value problems is presented. In this method the piecewise-uniform meshes are constructed and fitted to the boundary layer regions to adapt singular behavior of the operator in these narrow regions. Both the cases, layer on the left side boundary and layer on the right side boundary, are discussed. It is shown that the method composed of an upwind difference operator on the piecewise uniform mesh is parameter-uniform by establishing a robust error estimate. The effect of small delay on the boundary layer solution is shown by plotting the graphs of the solution for different delay values for several numerical examples. Numerical results in terms of maximum absolute error are tabulated to demonstrate the efficiency of the method.

Key Words.
fitted mesh, finite difference, singular perturbation, differential-difference equation, delay, boundary layer, action potential

AMS(MOS) Subject Classifications.
34K28, 34K26, 34K10

202 Numerical study of normal pressure distribution in entrance flow between parallel plates, I. Finite difference calculations. *Kenshu Shimomukai and Hidesada Kanda*.

Abstract.
This paper deals with the computation of flow between two parallel plates, including the pressure distribution in the entrance region. There are few implicit solutions available for the pressure distribution in the normal or y-direction of such flows. The pressure distribution in the y-direction is thus computed for the first time for flow between parallel plates. The minimum critical Reynolds number for laminar-turbulent transition is known to be in the range from 1300 to 1400, and we have thus focused our finite difference computations of the pressure gradient in the y-direction at Reynolds numbers (Re) between 100 and 5000. Our results have enabled us to conclude that a large difference in pressure between the wall and the centerline exists near the inlet for a low Re and decreased as Re increased. The pressure at the wall is lower than that in the central core for $Re \leq 5000$, indicating that the pressure distribution is contrary to Bernoulli's law across parallel plates, although the law does not apply to viscous flow.

Key Words.
computational fluid dynamics, numerical analysis.

AMS(MOS) Subject Classifications.
15A15, 15A09, 15A23

219 A study of the fast solution of the occluded radiosity equation. *Kendall Atkinson and David Chien*.

Abstract.
Consider the numerical solution of the radiosity equation over an occluded surface S using a collocation method based on piecewise polynomial interpolation. Hackbusch and Nowak [Numer. Math., 54 (1989) pp. 463–491] proposed a fast method of solution for boundary integral equations, and we consider the application of their method to the collocation solution of the radiosity equation. We give a combined analytical and experimental study of the 'clustering method' of Hackbusch and Nowak, yielding further insight into the method.

Key Words.
Radiosity equation, clustering method, collocation, fast solution method

AMS(MOS) Subject Classifications.
65R20, 65F99

251 Iterative sinc-convolution method for solving radiosity equation in computer graphics. *Ahmad Reza Naghsh-Nilchi and Shahram Daroee*.

Abstract.
A new and efficient sinc-convolution algorithm is introduced for the numerical solution of the radiosity equation. This equation has many applications including the production of photorealistic images. The method of sinc-convolution is based on using collocation to replace multi-dimensional convolution-type integrals—such as two dimensional radiosity integral equations—by a system of algebraic equations. The developed algorithm solves for the illumination of a surface or a set of surfaces when both reflectivity and emissivity of those surfaces are known. It separates the radiosity equation's variables to approximate its solution. The separation of variables allows the elimination of the formulation of huge full matrices and therefore reduces required storage, as well as computational complexity, as compared with classical approaches. Also, the highly singular nature of the kernel, which results in great difficulties using classical numerical methods, poses absolutely no difficulties using sinc-convolution. In addition, the new algorithm can be readily adapted for parallel computation for an even faster computational speed. The results show that the developed algorithm clearly reveals the color bleeding phenomenon which is a natural phenomenon not revealed by many other methods. These advantages should make real-time photorealistic image production feasible.

Key Words.
radiosity, sinc, sinc-convolution, photorealistic, computer graphics

AMS(MOS) Subject Classifications.
68U05, 65D18, 65D30

263 Fast multilevel evaluation of smooth radial basis function expansions. *Oren E. Livne and Grady B. Wright*.

Abstract.
Radial basis functions (RBFs) are a powerful tool for interpolating/approximating multidimensional scattered data. Notwithstanding, RBFs pose computational challenges, such as the efficient evaluation of an n-center RBF expansion at m points. A

direct summation requires $O(nm)$ operations. We present a new multilevel method whose cost is only $O((n + m)\ln(1/\delta)^d)$, where δ is the desired accuracy and d is the dimension. The method applies to smooth radial kernels, e.g., Gaussian, multiquadric, or inverse multiquadric. We present numerical results, discuss generalizations, and compare our method to other fast RBF evaluation methods. This multilevel summation algorithm can be also applied beyond RBFs, to discrete integral transform evaluation, Gaussian filtering and de-blurring of images, and particle force summation.

Key Words.
radial basis functions, fast multilevel multi-summation, integral transforms, particle interaction

AMS(MOS) Subject Classifications.
41A21, 41A30, 41A63, 65D25, 65N06, 65R10, 68Q25

Abstract.
A one dimensional singularly perturbed elliptic problem with discontinuous coefficients is considered. The domain under consideration is partitioned into two subdomains. In the first subdomain a convection-diffusion-reaction equation is posed. In the second one we have a pure reaction-diffusion equation. The problem is discretized using an inverse-monotone finite volume method on Shishkin meshes. We establish an almost second-order global pointwise convergence that is uniform with respect to the perturbation parameter. Numerical experiments that support the theoretical results are given.

Key Words.
convection-diffusion problems, singular perturbation, asymptotic analysis, finite volume methods, modified upwind approximations, uniform convergence, Shishkin mesh

AMS(MOS) Subject Classifications.
34A36,34E05, 34E15, 65L10, 65L12, 65L20, 65L50

Dedicated to the memory of Professor Aurel Cornea

Abstract.
Suppose G is a body in $\mathbb{R}^d$, $D \subset G$ is compact, and ρ a unit measure on ∂G. Inverse balayage refers to the question of whether there exists a measure ν supported inside D such that ρ and ν produce the same electrostatic field outside G. Establishing a duality principle between two extremal problems, it is shown that such an inverse balayage exists if and only if

$$\sup_{\mu} \left\{ \inf_{y \in D} U^{\mu}(y) - \int U^{\rho}\, d\mu \right\} = 0\,,$$

where the supremum is taken over all unit measures μ on ∂G and U^{μ} denotes the electrostatic potential of μ. A consequence is that pairs (ρ, D) admitting such an inverse balayage can be characterized by a ρ-mean-value principle, namely,

$$\sup_{z\in D} h(z) \geq \int h\, d\rho \geq \inf_{z\in D} h(z)$$

for all h harmonic in G and continuous up to the boundary.
In addition, two approaches for the construction of an inverse balayage related to extremal point methods are presented, and the results are applied to problems concerning the determination of restricted Chebychev constants in the theory of polynomial approximation.

Key Words.
logarithmic potential, Newtonian potential, balayage, inverse balayage, linear optimization, duality, Chebychev constant, extremal problem.

AMS(MOS) Subject Classifications.
31A15, 30C85, 41A17.

Abstract.
This paper derives a novel method of approximating the Hilbert transform by the use of sinc convolution. The proposed method may be used to approximate the Hilbert transform over any subinterval Γ of the real line $\mathbb{R} \equiv (-\infty, \infty)$, which means the interval Γ may be a finite or semi-infinite interval, or the entire real line $\mathbb{R}$. Given a column vector $\boldsymbol{f}$ consisting of m values of a function f defined on m sinc points of Γ, we obtain a column vector $\boldsymbol{g} = \boldsymbol{A}\boldsymbol{f}$ whose entries approximate the Hilbert transform on the same set of m sinc points. The present paper describes an explicit method for the construction of such a matrix $\boldsymbol{A}$.

Key Words.
sinc methods, Hilbert transform, Cauchy principal value integral

AMS(MOS) Subject Classifications.
65R10

Abstract.
A method to compute the eigenvalues of the spheroidal wave equations is proposed, as an application of a theorem on eigenvalues of certain classes of infinite matrices. The computation of its inverse problem (namely, solving another parameter c^2 for given eigenvalue λ) is likewise given. As a result, precise and explicit error estimates are obtained for the approximated eigenvalues.

Key Words.
spheroidal wave equation, eigenvalue, numerical computation, error estimate, infinite symmetric tridiagonal matrix

AMS(MOS) Subject Classifications.
34L16