

## A NETWORK PROGRAMMING APPROACH IN SOLVING DARCY'S EQUATIONS BY MIXED FINITE-ELEMENT METHODS\*

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**Abstract.** We use the null space algorithm approach to solve the augmented systems produced by the mixed finite-element approximation of Darcy's laws. Taking into account the properties of the graph representing the triangulation, we adapt the null space technique proposed in [M. ARIOLI AND L. BALDINI, *A backward error analysis of a null space algorithm in sparse quadratic programming*, SIAM J. Matrix Anal. and Applics., 23 (2001), pp. 425–442], where an iterative-direct hybrid method is described. In particular, we use network programming techniques to identify the renumbering of the triangles and the edges, which enables us to compute the null space without floating-point operations. Moreover, we extensively take advantage of the graph properties to build efficient preconditioners for the iterative algorithm. Finally, we present the results of several numerical tests.

**Key words.** augmented systems, sparse matrices, mixed finite-element, graph theory

**AMS subject classifications.** 65F05, 65F10, 64F25, 65F50, 65G05

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\*Received January 12, 2005. Accepted for publication September 5, 2005. Recommended by M. Benzi. The work of the first author was supported in part by EPSRC grants GR/R46641/01 and GR/S42170. The work of second author was supported by EPSRC grant GR/R46427/01 and partially by the CNR Short-Term Mobility Programme, 2005.

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