Electronic Transactions on Numerical Analysis.
Volume 22, pp. 17-40, 2006.

ISSN 1068-9613.

ETNA
Kent State University
etna@mcs.kent.edu

DUAL VARIABLE METHODS FOR MIXED-HYBRID FINITE ELEMENT APPROXIMATION OF THE POTENTIAL FLUID FLOW PROBLEM IN POROUS MEDIA[*]

M. ARIOLI[†], J. MARYŠKA[‡], M. ROZLOŽNÍK[‡], AND M. TŮMA[‡]

Abstract. Mixed-hybrid finite element discretization of Darcy's law and the continuity equation that describe the potential fluid flow problem in porous media leads to symmetric indefinite saddle-point problems. In this paper we consider solution techniques based on the computation of a null-space basis of the whole or of a part of the left lower off-diagonal block in the system matrix and on the subsequent iterative solution of a projected system. This approach is mainly motivated by the need to solve a sequence of such systems with the same mesh but different material properties. A fundamental cycle null-space basis of the whole off-diagonal block is constructed using the spanning tree of an associated graph. It is shown that such a basis may be theoretically rather ill-conditioned. Alternatively, the orthogonal null-space basis of the sub-block used to enforce continuity over faces can be easily constructed. In the former case, the resulting projected system is symmetric positive definite and so the conjugate gradient method can be applied. The projected system in the latter case remains indefinite and the preconditioned minimal residual method (or the smoothed conjugate gradient method) should be used. The theoretical rate of convergence for both algorithms is discussed and their efficiency is compared in numerical experiments.

Key words. saddle-point problem, preconditioned iterative methods, sparse matrices, finite element method

AMS subject classifications. 65F05, 65F50

[*]Received January 3, 2005. Accepted for publication September 15, 2005. Recommended by M. Benzi. This work was supported by the project 1ET400300415 within the National Program of Research "Information Society" and by the project No. IAA1030405 by GA AS CR. The work of the first author was supported in part by EPSRC grants GR/R46641/01 and GR/S42170.

[†]Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, OX11 0QX, UK (M.Arioli@rl.ac.uk).

[‡]Institute of Computer Science, Academy of Sciences of the Czech Republic, Pod vodárenskou věží 2, 182 07 Prague 8, Czech Republic and Technical University of Liberec, Department of Modelling of Processes, Hálkova 6, CZ-461 17 Liberec, Czech Republic (jiri.maryska@vslib.cz, miro@cs.cas.cz, tuma@cs.cas.cz).