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PROBING METHODS FOR SADDLE-POINT PROBLEMS*

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Abstract. Several Schur complement-based preconditioners have been proposed for solving (generalized) saddle-point problems. We consider matrices where the Schur complement has rapid decay over some graph known *a priori*. This occurs for many matrices arising from the discretization of systems of partial differential equations, and this graph is then related to the mesh. We propose the use of probing methods to approximate these Schur complements in preconditioners for saddle-point problems. We demonstrate these techniques for the block-diagonal and constraint preconditioners proposed by [Murphy, Golub and Wathen '00], [de Sturler and Liesen '04] and [Siefert and de Sturler '05]. However, these techniques are applicable to many other preconditioners as well. We discuss the implementation of probing methods, and we consider the application of those approximations in preconditioners for Navier-Stokes problems and metal deformation problems. Finally, we study eigenvalue clustering for the preconditioned matrices, and we present convergence and timing results for various problem sizes. These results demonstrate the effectiveness of the proposed preconditioners with probing-based approximate Schur complements.

Key words. saddle-point systems, constraint preconditioners, Krylov methods, Schur complements, probing

AMS subject classifications. 65F10, 65F50, 05C15

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