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COMBINATORIAL ALGORITHMS FOR COMPUTING COLUMN SPACE BASES THAT HAVE SPARSE INVERSES*

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Abstract. This paper presents a new combinatorial approach towards constructing a sparse, implicit basis for the null space of a sparse, under-determined matrix A. Our approach is to compute a column space basis of A that has a sparse inverse, which could be used to represent a null space basis in implicit form. We investigate three different algorithms for computing column space bases: two greedy algorithms implemented using graph matchings, and a third, which employs a divide and conquer strategy implemented with hypergraph partitioning followed by a matching. Our results show that for many matrices from linear programming, structural analysis, and circuit simulation, it is possible to compute column space bases having sparse inverses, contrary to conventional wisdom. The hypergraph partitioning method yields sparser basis inverses and has low computational time requirements, relative to the greedy approaches. We also discuss the complexity of selecting a column space basis when it is known that such a basis exists in block diagonal form with a given small block size.

Key words. sparse column space basis, sparse null space basis, block angular matrix, block diagonal matrix, matching, hypergraph partitioning, inverse of a basis

AMS subject classifications. 65F50, 68R10, 90C20

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