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OBTAINING BOUNDS ON THE TWO NORM OF A MATRIX FROM THE SPLITTING LEMMA*

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Dedicated to Alan George on the occasion of his 60th birthday

Abstract. The splitting lemma is one of the two main tools of *support theory*, a framework for bounding the condition number of definite and semidefinite preconditioned linear systems. The splitting lemma allows the analysis of a complicated system to be partitioned into analyses of simpler systems. The other tool is the symmetric-product-support lemma, which provides an explicit spectral bound on a preconditioned matrix.

The symmetric-product-support lemma shows that under suitable conditions on the null spaces of A and B, the finite eigenvalues of the pencil (A, B) are bounded by $\|W\|_2^2$, where $U = VW$, $A = UU^T$, and $B = VV^T$. To apply the lemma, one has to construct a W satisfying these conditions, and to bound its 2-norm.

In this paper we show that in all its existing applications, the splitting lemma can be viewed as a mechanism to bound $\|W\|_2^2$ for a given W. We also show that this bound is sometimes tighter than other easily-computed bounds on $\|W\|_2^2$, such as $\|W\|_F^2$ and $\|W\|_1 \|W\|_\infty$. The paper shows that certain regular splittings have useful algebraic and combinatorial interpretations. In particular, we derive six separate algebraic bounds on the 2-norm of a real matrix; to the best of our knowledge, these bounds are new.

Key words. matrix norm bounds, two-norm, norm bounds for sparse matrices, splitting lemma, support theory, support preconditioning

AMS subject classifications. 15A60, 65F10, 65F35, 65F50

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