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ETNA
Kent State University
etna@mcs.kent.edu

QUADRATURE OVER THE SPHERE *

KENDALL ATKINSON† AND ALVISE SOMMARIVA‡

Abstract. Consider integration over the unit sphere in $\mathbb{R}^3$, especially when the integrand has singular behaviour in a polar region. In an earlier paper [4], a numerical integration method was proposed that uses a transformation that leads to an integration problem over the unit sphere with an integrand that is much smoother in the polar regions of the sphere. The transformation uses a *grading parameter* q. The trapezoidal rule is applied to the spherical coordinates representation of the transformed problem. The method is simple to apply, and it was shown in [4] to have convergence $O\left(h^{2q}\right)$ or better for integer values of $2q$. In this paper, we extend those results to non-integral values of $2q$. We also examine superconvergence that was observed when $2q$ is an odd integer. The overall results agree with those of [11], although the latter is for a different, but related, class of transformations.

Key words. spherical integration, trapezoidal rule, Euler-MacLaurin expansion

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†Depts of Mathematics and Computer Science, University of Iowa

‡Dept of Pure and Applied Mathematics, University of Padua. Supported by the research project CPDA028291 ”Efficient approximation methods for nonlocal discrete transform” of the University of Padua.