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FAST GIVENS TRANSFORMATION FOR QUATERNION VALUED MATRICES APPLIED TO HESSENBERG REDUCTIONS*

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Abstract. In a previous paper we investigated Givens transformations applied to quaternion valued matrices. Since arithmetic operations with quaternions are very costly it is desirable to reduce the number of arithmetic operations with quaternions. We show that the Fast Givens transformation, known for the real case, can also be defined for quaternion valued matrices, and we apply this technique to the reduction of an arbitrary quaternion valued matrix to upper Hessenberg form and also include a numerical example. We offer two algorithms. One is based on the classical real case using dynamically two transformation matrices, while the other is based on four transformation matrices where in each step that matrix is selected that has the smallest condition number. For the first algorithm we show that the essential information (namely the two numbers s and c which define the Givens transformation) can be stored in only one variable. This is apparently even new for the real case. We include, necessarily, some investigations on the determination of the relevant condition numbers. We show that in general the application of the Fast Givens transformation in the quaternion case is not as favorable as in the real case with respect to (relative) savings in arithmetic operations. We begin with some introduction into the field of quaternions. In the end in an appendix we present some results concerning the computation of roots of quaternions which in some cases are needed.

Key words. Fast Givens rotation, quaternions, quaternion valued matrices, Hessenberg form for quaternion valued matrices, roots of quaternions.

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