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MINIMIZATION PROPERTIES AND SHORT RECURRENCES FOR KRYLOV SUBSPACE METHODS*

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Dedicated to Wilhelm Niethammer on the occasion of his 60th birthday.

Abstract. It is well known that generalized conjugate gradient (cg) methods, fulfilling a minimization property in the whole spanned Krylov space, cannot be formulated with short recurrences for nonsymmetric system matrices. Here, Krylov subspace methods are proposed that do fulfill a minimization property and can be implemented as short recurrence method at the same time. These properties are achieved by a generalization of the cg concept. The convergence and the geometric behavior of these methods are investigated.

Practical applications show that first realizations of these methods are already competitive with commonly used techniques such as smoothed biconjugate gradients or QMR. Better results seem to be possible by further improvements of the techniques. However, the purpose of this paper is not to propagate a special method, but to stimulate research and development of new iterative linear solvers.

Key words. conjugate gradients, convergence, linear systems, Krylov methods.

AMS subject classifications. 65F10, 65F50, 40A05.

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