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ETNA
Kent State University
etna@mcs.kent.edu

DISPLACEMENT PRECONDITIONER FOR TOEPLITZ LEAST SQUARES ITERATIONS *

RAYMOND H. CHAN †, JAMES G. NAGY ‡, AND ROBERT J. PLEMMONS §

Abstract. We consider the solution of least squares problems $\min||b - Ax||_2$ by the preconditioned conjugate gradient (PCG) method, for $m \times n$ complex Toeplitz matrices A of rank n. A circulant preconditioner C is derived using the T. Chan optimal preconditioner for $n \times n$ matrices using the displacement representation of A^*A. This allows the fast Fourier transform (FFT) to be used throughout the computations, for high numerical efficiency. Of course A^*A need never be formed explicitly. Displacement–based preconditioners have also been shown to be very effective in linear estimation and adaptive filtering. For Toeplitz matrices A that are generated by 2π-periodic continuous complex-valued functions without any zeros, we prove that the singular values of the preconditioned matrix AC^{-1} are clustered around 1, for sufficiently large n. We show that if the condition number of A is of $O(n^\alpha)$, $\alpha > 0$, then the least squares conjugate gradient method converges in at most $O(\alpha \log n + 1)$ steps. Since each iteration requires only $O(m \log n)$ operations using the FFT, it follows that the total complexity of the algorithm is then only $O(\alpha m \log^2 n + m \log n)$. Conditions for *superlinear convergence* are given and numerical examples are provided illustrating the effectiveness of our methods.

Key words. circulant preconditioner, conjugate gradient,displacement representation, fast Fourier transform (FFT), Toeplitz operator.

AMS subject classifications. 65F10, 65F15.

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‡Department of Mathematics, Southern Methodist University, Dallas, TX 75275-0156. This research was supported by Oak Ridge Associated Universities under grant no. 009707.

§ Department of Mathematics and Computer Science, Wake Forest University, P.O. Box 7388, Winston-Salem, NC 27109. This research was supported by the US Air Force under grant no. AFOSR-91-0163 and NSF grant no. CCR-92-01105.