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LOCALIZED POLYNOMIAL BASES ON THE SPHERE*

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Abstract. The subject of many areas of investigation, such as meteorology or crystallography, is the reconstruction of a continuous signal on the 2-sphere from scattered data. A classical approximation method is *polynomial interpolation*. Let V_n denote the space of polynomials of degree at most n on the unit sphere $\mathbb{S}^2 \subset \mathbb{R}^3$. As it is well known, the so-called *spherical harmonics* form an orthonormal basis of the space V_n. Since these functions exhibit a poor localization behavior, it is natural to ask for better localized bases. Given $\{\xi_i\}_{i=1,\ldots,(n+1)^2} \subset \mathbb{S}^2$, we consider the spherical polynomials

$$\varphi_i^n(\xi) := \sum_{l=0}^{n} \frac{2l+1}{4\pi} P_l(\xi_i \cdot \xi),$$

where P_l denotes the Legendre polynomial of degree l normalized according to the condition $P_l(1) = 1$. In this paper, we present systems of $(n+1)^2$ points on $\mathbb{S}^2$ that yield localized polynomial bases of the above form.

Key words. fundamental systems, localization, matrix condition, reproducing kernel.

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