Electronic Transactions on Numerical Analysis.
Volume 19, pp. 58-83, 2005.

ISSN 1068-9613.

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ASYMPTOTIC APPROXIMATIONS OF INTEGRALS: AN INTRODUCTION, WITH RECENT DEVELOPMENTS AND APPLICATIONS TO ORTHOGONAL POLYNOMIALS*

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Abstract. In the first part we discuss the concept of asymptotic expansion and its importance in applications. We focus our attention on special functions defined through integrals and consider their approximation by means of asymptotic expansions. We explain the general ideas of the theory of asymptotic expansions of integrals and describe two classical methods (Watson's lemma and the saddle point method) and modern methods (distributional methods). In the second part we apply these ideas to approximate (in an asymptotic sense) polynomials of the Askey table in terms of simpler polynomials of the Askey table. We consider two different types of asymptotic expansions that have been recently developed: i) some parameter of the polynomial is large or ii) the degree (and perhaps the variable too) of the polynomial is large. For each situation we employ a different asymptotic method. In the first case we use the technique of "matching of the generating functions at the origin". In the second one we employ a modified version of the saddle point method together with the theory of two-point Taylor expansions. In the first case the asymptotic expansion results in a finite sum of polynomials. In the second one the asymptotic expansion is a convergent infinite series of polynomials. We conclude the paper with information on other recent developments in the research on asymptotic expansions of integrals.

Key words. Asymptotic expansions of integrals, asymptotics of orthogonal polynomials.

AMS subject classifications. 41A60, 33C65.

*Received November 30, 2002. Accepted for publication September 5, 2003. Communicated by R. Alvarez-Nodarse.

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