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Contents

Abstract.
In this paper we study the orthogonality conditions satisfied by Jacobi polynomials $P_n^{(\alpha,\beta)}$ when the parameters α and β are not necessarily > -1. We establish orthogonality on a generic closed contour on a Riemann surface. Depending on the parameters, this leads to either full orthogonality conditions on a single contour in the plane, or to multiple orthogonality conditions on a number of contours in the plane. In all cases we show that the orthogonality conditions characterize the Jacobi polynomial $P_n^{(\alpha,\beta)}$ of degree n up to a constant factor.

Key Words.
Jacobi polynomials, orthogonality, Rodrigues formula, zeros.

AMS(MOS) Subject Classifications.
33C45.

Files.
vol.19.2005/pp1-17.dir/pp1-17.ps; vol.19.2005/pp1-17.dir/pp1-17.pdf;

Forward References.

Abstract.
In this paper we study the asymptotic behavior of sequences of Heine-Stieltjes and Van Vleck polynomials for a class of generalized Lamé differential equations connected with certain equilibrium problems on the unit circle.

Key Words.
Heine-Stieltjes polynomials, Van Vleck polynomials, zeros, asymptotics.

AMS(MOS) Subject Classifications.
33C45.

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vol.19.2005/pp18-28.dir/pp18-28.ps; vol.19.2005/pp18-28.dir/pp18-28.pdf;

Forward References.

Abstract.
In this paper, we give strong asymptotics of extremal polynomials with respect to

varying measures of the form $d\sigma_n = \frac{d\sigma}{|Y_n|^p}$, where σ is a positive measure on a closed analytic Jordan curve C, and $\{Y_n\}$ is a sequence of polynomials such that for each n, Y_n has exactly degree n and all its zeros $(\alpha_{n,i})$, $i = 1, 2, \ldots$, lie in the exterior of C.

Key Words.
Rational Approximation, Orthogonal Polynomials, Varying Measures.

AMS(MOS) Subject Classifications.
30E10, 41A20, 42C05.

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vol.19.2005/pp29-36.dir/pp29-36.ps; vol.19.2005/pp29-36.dir/pp29-36.pdf;

Forward References.

37 An electrostatic interpretation of the zeros of the Freud-type orthogonal polynomials. *A. Garrido, J. Arvesú, and F. Marcellán.*

Abstract.
Polynomials orthogonal with respect to a perturbation of the Freud weight function by the addition of a mass point at zero are considered. These polynomials, called Freud-type orthogonal polynomials, satisfy a second order linear differential equation with varying polynomial coefficients. It plays an important role in the electrostatic interpretation for the distribution of zeros of the corresponding orthogonal polynomials.

Key Words.
Freud weights, orthogonal polynomials, zeros, potential theory, semiclassical linear functional.

AMS(MOS) Subject Classifications.
65F05.33C45, 42C05.

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vol.19.2005/pp37-47.dir/pp37-47.ps; vol.19.2005/pp37-47.dir/pp37-47.pdf;

Forward References.

48 Szegő quadrature and frequency analysis. *Leyla Daruis, Olav NjÅstad, and Walter Van Assche.*

Abstract.
A series of papers have treated the frequency analysis problem by studying the zeros of orthogonal polynomials on the unit circle with respect to measures determined by observations of the signal. In the recent paper [3], a different approach was used, where properties of Szegő quadrature formulas associated with the zeros of para-orthogonal polynomials with respect to the same measures were used to determine the frequencies and amplitudes in the signal. In this paper we carry this approach further, and obtain more conclusive results.

Key Words.
Szegő polynomials, Szegő quadrature formula, frequency analysis problem.

AMS(MOS) Subject Classifications.
41A55, 33C45.

Files.
vol.19.2005/pp48-57.dir/pp48-57.ps; vol.19.2005/pp48-57.dir/pp48-57.pdf;

Forward References.

58 Asymptotic approximations of integrals: an introduction, with recent developments and applications to orthogonal polynomials. *Chelo Ferreira, José L. López, Esmeralda Mainar, and Nico M. Temme.*

Abstract.
In the first part we discuss the concept of asymptotic expansion and its importance in applications. We focus our attention on special functions defined through integrals and consider their approximation by means of asymptotic expansions. We explain the general ideas of the theory of asymptotic expansions of integrals and describe two classical methods (Watson's lemma and the saddle point method) and modern methods (distributional methods). In the second part we apply these ideas to approximate (in an asymptotic sense) polynomials of the Askey table in terms of simpler polynomials of the Askey table. We consider two different types of asymptotic expansions that have been recently developed: i) some parameter of the polynomial is large or ii) the degree (and perhaps the variable too) of the polynomial is large. For each situation we employ a different asymptotic method. In the first case we use the technique of "matching of the generating functions at the origin". In the second one we employ a modified version of the saddle point method together with the theory of two-point Taylor expansions. In the first case the asymptotic expansion results in a finite sum of polynomials. In the second one the asymptotic expansion is a convergent infinite series of polynomials. We conclude the paper with information on other recent developments in the research on asymptotic expansions of integrals.

Key Words.
Asymptotic expansions of integrals, asymptotics of orthogonal polynomials.

AMS(MOS) Subject Classifications.
41A60, 33C65.

Files.
vol.19.2005/pp58-83.dir/pp58-83.ps; vol.19.2005/pp58-83.dir/pp58-83.pdf;

Forward References.

84 Localized Polynomial Bases on the Sphere. *Noemí Laín Fernández.*

Abstract.
The subject of many areas of investigation, such as meteorology or crystallography, is the reconstruction of a continuous signal on the 2-sphere from scattered data. A classical approximation method is *polynomial interpolation*. Let V_n denote the space of polynomials of degree at most n on the unit sphere $\mathbb{S}^{\nvDash} \subset \mathbb{R}^{\nVdash}$. As it is well known, the so-called *spherical harmonics* form an orthonormal basis of the space V_n. Since these functions exhibit a poor localization behavior, it is natural to ask for better localized bases. Given $\{\xi_i\}_{i=1,\dots,(n+1)^2} \subset \mathbb{S}^{\nvDash}$, we consider the spherical polynomials

$$\varphi_i^n(\xi) := \sum_{l=0}^{n} \frac{2l+1}{4\pi} P_l(\xi_i \cdot \xi),$$

where P_l denotes the Legendre polynomial of degree l normalized according to the condition $P_l(1)=1$. In this paper, we present systems of $(n+1)^2$ points on $\mathbb{S}^{\nvDash}$ that yield localized polynomial bases of the above form.

Key Words.
fundamental systems, localization, matrix condition, reproducing kernel.

AMS(MOS) Subject Classifications.
41A05, 65D05, 15A12.

Files.
vol.19.2005/pp84-93.dir/pp84-93.ps; vol.19.2005/pp84-93.dir/pp84-93.pdf;

Forward References.

Abstract.
In this paper we consider functions from a Besov-Nikol'skiĭ class. We give constructive characteristics of this class. We establish criteria for a function to be in this Besov-Nikol'skiĭ class by means of conditions on its Fourier coefficients. We also discuss embedding theorems between some classes of functions.

Key Words.
Moduli of smoothness, Besov-Nikol'skiĭ class, Best approximation, Fourier coefficients, Embedding theorems.

AMS(MOS) Subject Classifications.
42A10, 42A16, 41A50, 42A32, 42A50.

Files.
vol.19.2005/pp94-104.dir/pp94-104.ps; vol.19.2005/pp94-104.dir/pp94-104.pdf;

Forward References.

Abstract.
In this work we prove that Hessenberg's infinite matrix, associated with an hermitian OPS that generalizes the Jacobi matrix, is normal under the assumption that the OPS is generated from a discrete infinite bounded distribution of non-aligned points in the complex plane with some geometrical restrictions. This matrix is also normal if we consider a real bounded distribution with a finite amount of atomic complex points. In this case we still have normality with infinite points, but an additional condition is required. Some other interesting properties of that matrix are obtained.

Key Words.
orthogonal polynomials, Hessenberg's matrix, normal operator.

AMS(MOS) Subject Classifications.
33C45.

Files.
vol.19.2005/pp105-112.dir/pp105-112.ps;
vol.19.2005/pp105-112.dir/pp105-112.pdf;

Forward References.

113 Orthogonal Laurent polynomials and quadratures on the unit circle and the real half-line. *Ruymán Cruz-Barroso, and Pablo González-Vera.*

Abstract.
The purpose of this paper is the computation of quadrature formulas based on Laurent polynomials in two particular situations: the Real Half-Line and the Unit Circle. Comparative results and a connection with the split Levinson algorithm are established. Illustrative numerical examples are approximate integrals of the form

$$\int_{-1}^{1} \frac{f(x)}{(x+\lambda)^r} \omega(x)\, dx \quad , \quad r = 1, 2, 3, \ldots$$

with $f(x)$ a continuous function on $[-1, 1]$, $\omega(x) \geq 0$ a weight function on this interval and $\lambda \in \mathbb{R}$ such that $|\lambda| > 1$ is required. Here the classical Gaussian quadrature is an extremely slow procedure.

Key Words.
orthogonal Laurent polynomials, L-Gaussian quadrature, Szegö quadrature, three-term recurrence relations, split Levinson algorithm, numerical quadrature.

AMS(MOS) Subject Classifications.
41A55, 33C45, 65D30.

Files.
vol.19.2005/pp113-134.dir/pp113-134.ps;
vol.19.2005/pp113-134.dir/pp113-134.pdf;

Forward References.