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SOME THEORETICAL RESULTS DERIVED FROM POLYNOMIAL NUMERICAL HULLS OF JORDAN BLOCKS*

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Abstract. The polynomial numerical hull of degree k for a square matrix A is a set in the complex plane designed to give useful information about the norms of functions of the matrix; it is defined as

$$\{z \in \mathbb{C} : \ \|p(A)\| \geq |p(z)| \ \text{ for all polynomials } p \text{ of degree } k \text{ or less}\}.$$

In a previous paper [V. Faber, A. Greenbaum, and D. Marshall, *The polynomial numerical hulls of Jordan blocks and related matrices*, Linear Algebra Appl., 374 (2003), pp. 231–246] analytic expressions were derived for the polynomial numerical hulls of Jordan blocks. In this paper, we explore some consequences of these results. We derive lower bounds on the norms of functions of Jordan blocks and triangular Toeplitz matrices that approach equalities as the matrix size approaches infinity. We demonstrate that even for moderate size matrices these bounds give fairly good estimates of the behavior of matrix powers, the matrix exponential, and the resolvent norm. We give new estimates of the convergence rate of the GMRES algorithm applied to a Jordan block. We also derive a new estimate for the field of values of a general Toeplitz matrix.

Key words. polynomial numerical hull, field of values, Toeplitz matrix.

AMS subject classifications. 15A60, 65F15, 65F35.

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