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# STABILITY AND SENSITIVITY OF DARBOUX TRANSFORMATION WITHOUT PARAMETER *

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**Abstract.** The monic Jacobi matrix is a tridiagonal matrix which contains the parameters of the three-term recurrence relation satisfied by the sequence of monic polynomials orthogonal with respect to a measure. Darboux transformation without parameter changes a monic Jacobi matrix associated with a measure $\mu$ into the monic Jacobi matrix associated with $xd\mu$. This transformation has been used in several numerical problems as in the computation of Gaussian quadrature rules. In this paper, we analyze the stability of an algorithm which implements Darboux transformation without parameter numerically and we also study the sensitivity of the problem. The main result of the paper is that, although the algorithm for Darboux transformation without parameter is not backward stable, it is forward stable. This means that the forward errors are of similar magnitude to those produced by a backward stable algorithm. Moreover, bounds for the forward errors computable with low cost are presented. We also apply the results to some classical families of orthogonal polynomials.

**Key words.** Darboux transformation, orthogonal polynomials, stability, sensitivity, tridiagonal matrices, $LU$ factorization, $LR$ algorithm.

**AMS subject classifications.** 65G50, 42C05, 15A23, 65F30, 65F35.

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