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COLLOCATION METHODS FOR CAUCHY SINGULAR INTEGRAL EQUATIONS ON THE INTERVAL*

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Abstract. In this paper we consider polynomial collocation methods for the numerical solution of a singular integral equation over the interval, where the operator of the equation is supposed to be of the form $aI+b\mu^{-1}S\mu I+K$ with S the Cauchy singular integral operator, with piecewise continuous coefficients a and b, and with a Jacobi weight μ. K denotes an integral operator with a continuous kernel function. To the integral equation we apply two collocation methods, where the collocation points are the Chebyshev nodes of the first and second kind and where the trial space is the space of polynomials multiplied by another Jacobi weight. For the stability and convergence of this collocation scheme in weighted L^2-spaces, we derive necessary and sufficient conditions. Moreover, we discuss stability of operator sequences belonging to algebras generated by the sequences of the collocation methods for the above described operators. Finally, the so-called splitting property of the singular values of the sequences of the matrices of the discretized equations is proved.

Key words. Cauchy singular integral equation, polynomial collocation method, stability, singular values, splitting property.

AMS subject classifications. 45L10, 65R20, 65N38.

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