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ON THE EXISTENCE THEOREMS OF KANTOROVICH, MIRANDA AND BORSUK*

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Abstract. The theorems of Kantorovich, Miranda and Borsuk all give conditions on the existence of a zero of a nonlinear mapping. In this paper we are concerned with relations between these theorems in terms of generality in the case that the mapping is finite-dimensional. To this purpose we formulate a generalization of Miranda's theorem, holding for arbitrary norms instead of just the l_∞-norm. As our main results we then prove that the Kantorovich theorem reduces to a special case of this generalized Miranda theorem as well as to a special case of Borsuk's theorem. Moreover, it turns out that, essentially, the Miranda theorems are themselves special cases of Borsuk's theorem.

Key words. nonlinear equations, existence theorems, fixed points, Newton-Kantorovich theorem, Miranda theorem, Borsuk theorem.

AMS subject classifications. 47H10, 47J05, 65H10.

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