Electronic Transactions on Numerical Analysis.
Volume 16, pp. 30-49, 2003.
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ISSN 1068-9613.

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VAIDYA'S PRECONDITIONERS: IMPLEMENTATION AND EXPERIMENTAL STUDY*

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Abstract. We describe the implementation and performance of a novel class of preconditioners. These preconditioners were proposed and theoretically analyzed by Pravin Vaidya in 1991, but no report on their implementation or performance in practice has ever been published. We show experimentally that these preconditioners have some remarkable properties. We show that within the class of diagonally-dominant symmetric matrices, the cost and convergence of these preconditioners depends almost only on the nonzero structure of the matrix, but not on its numerical values. In particular, this property leads to robust convergence behavior on difficult 3-dimensional problems that cause stagnation in incomplete-Cholesky preconditioners (more specifically, in drop-tolerance incomplete Cholesky without diagonal modification, with diagonal modification, and with relaxed diagonal modification). On such problems, we have observed cases in which a Vaidya-preconditioned solver is more than 6 times faster than an incomplete-Cholesky-preconditioned solver, when we allow similar amounts of fill in the factors of both preconditioners. We also show that Vaidya's preconditioners perform and scale similarly or better than drop-tolerance relaxed-modified incomplete Cholesky preconditioners on a wide range of 2-dimensional problems. In particular, on anisotropic 2D problems, Vaidya's preconditioners deliver robust convergence independently of the direction of anisotropy and the ordering of the unknowns. However, on many 3D problems in which incomplete-Cholesky-preconditioned solvers converge without stagnating, Vaidya-preconditioned solvers are much slower. We also show how the insights gained from this study can be used to design faster and more robust solvers for some difficult problems.

Key words. linear-equation solvers, iterative solvers, preconditioning, support preconditioning, support theory, maximum-spanning trees, experimental study.

AMS subject classifications. 65-05, 65F10, 65F35, 65F50, 65N22, 05C05, 05C50, 05C85.

*Received September 2, 2001. Accepted for publication January 3, 2003. Recommended by Tony Chan.
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