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EFFICIENT SOLUTION OF SYMMETRIC EIGENVALUE PROBLEMS USING MULTIGRID PRECONDITIONERS IN THE LOCALLY OPTIMAL BLOCK CONJUGATE GRADIENT METHOD. *

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Abstract. We present a short survey of multigrid–based solvers for symmetric eigenvalue problems. We concentrate our attention on "off the shelf" and "black box" methods, which should allow solving eigenvalue problems with minimal, or no, effort on the part of the developer, taking advantage of already existing algorithms and software. We consider a class of such methods, where the multigrid only appears as a black-box tool for constructing the preconditioner of the stiffness matrix, and the base iterative algorithm is one of well-known off-the-shelf preconditioned gradient methods such as the locally optimal block preconditioned conjugate gradient method. We review some known theoretical results for preconditioned gradient methods that guarantee the optimal, with respect to the grid size, convergence speed. Finally, we present results of numerical tests, which demonstrate practical effectiveness of our approach for the locally optimal block conjugate gradient method preconditioned by the standard V-cycle multigrid applied to the stiffness matrix.

Key words. symmetric eigenvalue problems, multigrid preconditioning, preconditioned conjugate gradient iterative method

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