

A MULTIGRID METHOD FOR DISTRIBUTED PARAMETER ESTIMATION PROBLEMS*

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Abstract.

This paper considers problems of distributed parameter estimation from data measurements on solutions of partial differential equations (PDEs). A nonlinear least squares functional is minimized to approximately recover the sought parameter function (i.e., the model). This functional consists of a data fitting term, involving the solution of a finite volume or finite element discretization of the forward differential equation, and a Tikhonov-type regularization term, involving the discretization of a mix of model derivatives.

We develop a multigrid method for the resulting constrained optimization problem. The method directly addresses the discretized PDE system which defines a critical point of the Lagrangian. The discretization is cell-based. This system is strongly coupled when the regularization parameter is small. Moreover, the compactness of the discretization scheme does not necessarily follow from compact discretizations of the forward model and of the regularization term. We therefore employ a Marquardt-type modification on coarser grids. Alternatively, fewer grids are used and a preconditioned Krylov-space method is utilized on the coarsest grid. A collective point relaxation method (weighted Jacobi or a Gauss-Seidel variant) is used for smoothing. We demonstrate the efficiency of our method on a classical model problem from hydrology.

Key words. distributed parameter estimation; inverse problem; multigrid method.

AMS subject classifications. 65M32, 65N55.

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