L-CURVE CURVATURE BOUNDS
VIA LANCZOS BIDIAGONALIZATION *

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Abstract. The L-curve is often applied to determine a suitable value of the regularization parameter when solving ill-conditioned linear systems of equations with a right-hand side contaminated by errors of unknown norm. The location of the vertex of the L-curve typically yields a suitable value of the regularization parameter. However, the computation of the L-curve and of its curvature is quite costly for large problems; the determination of a point on the L-curve requires that both the norm of the regularized approximate solution and the norm of the corresponding residual vector be available. Recently, the L-ribbon, which contains the L-curve in its interior, has been shown to be suitable for the determination of the regularization parameter for large-scale problems. In this paper we describe how techniques similar to those employed for the computation of the L-ribbon can be used to compute a "curvature-ribbon," which contains the graph of the curvature of the L-curve. Both curvature- and L-ribbon can be computed fairly inexpensively by partial Lanczos bidiagonalization of the matrix of the given linear system of equations. A suitable value of the regularization parameter is then determined from these ribbons, and we show that an associated approximate solution of the linear system can be computed with little additional work.

Key words. Ill-posed problem, regularization, L-curve, Gauss quadrature.

AMS subject classifications. Ill-posed problem, regularization, L-curve, Gauss quadrature.