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ASYMPTOTICS FOR QUADRATIC HERMITE-PADÉ POLYNOMIALS ASSOCIATED WITH THE EXPONENTIAL FUNCTION *

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Abstract. The asymptotic behavior of quadratic Hermite-Padé polynomials $p_n, q_n, r_n \in \mathcal{P}_n$ of type I and $\mathfrak{p}_n$, $\mathfrak{q}_n$, $\mathfrak{r}_n \in \mathcal{P}_{2n}$ of type II associated with the exponential function are studied. In the introduction the background of the definition of Hermite-Padé polynomials is reviewed. The quadratic Hermite-Padé polynomials $p_n, q_n, r_n \in \mathcal{P}_n$ of type I are defined by the relation

$$p_n(z) + q_n(z)e^z + r_n(z)e^{2z} \ = \ O(z^{3n+2}) \quad \text{as} \quad z \to 0,$$

and the polynomials $\mathfrak{p}_n$, $\mathfrak{q}_n$, $\mathfrak{r}_n \in \mathcal{P}_{2n}$ of type II by the two relations

$$\mathfrak{p}_n(z)e^z - \mathfrak{q}_n(z) \ = \ O(z^{3n+1}) \quad \text{as} \quad z \to 0,$$
$$\mathfrak{p}_n(z)e^{2z} - \mathfrak{r}_n(z) \ = \ O(z^{3n+1}) \quad \text{as} \quad z \to 0.$$

Analytic descriptions are given for the arcs, on which the contracted zeros of both sets of the polynomials $\{p_n, q_n, r_n\}$ and $\{\mathfrak{p}_n, \mathfrak{q}_n, \mathfrak{r}_n\}$ cluster as $n \to \infty$. Analytic expressions are also given for the density functions of the asymptotic distributions of these zeros.

The description is based on an algebraic function of third degree and a harmonic function defined on the Riemann surface, which is associated with the algebraic function. The existence and basic properties of the asymptotic distributions of the zeros and the arcs on which these distributions live are proved, the asymptotic relations themselves are only conjectured. Numerical calculations are presented, which demonstrate the plausibility of these conjectures.

Key words. Quadratic Hermite-Padé polynomials of type I and type II, the exponential function, German and Latin polynomials, Hermite-Padé approximants.

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