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NUMERICAL CONDITION OF POLYNOMIALS IN DIFFERENT FORMS*

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Abstract. The zeros of high-degree polynomials are notoriously sensitive to changes in the coefficients, causing problems for available zero-finding software. In this paper, we study how this sensitivity depends on the polynomial representation. We first extend the algebraic characterization of polynomial pseudozero sets from the power basis to general bases. We show that for a polynomial, the numerical conditions of its values and zeros are closely related and can be visualized simultaneously by its pseudozero sets. Comparing the pseudozero sets on a set of testing polynomials in the power, Taylor, Chebyshev, and Bernstein bases reveals that appropriate representation of polynomials gives rise to locally well-conditioned zeros, which then leads to an Iterative Refinement Algorithm that combines symbolic formulation with numeric processing to reduce computational errors of polynomial zeros located in the region of interest.

Key words. polynomial equation, polynomial basis, pseudozero set, iterative refinement algorithm.

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