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ETNA
Kent State University
etna@mcs.kent.edu

ERROR ANALYSIS OF QR ALGORITHMS FOR COMPUTING LYAPUNOV EXPONENTS[*]

EDWARD J. MCDONALD[†] AND DESMOND J. HIGHAM[‡]

Abstract. Lyapunov exponents give valuable information about long term dynamics. The discrete and continuous QR algorithms are widely used numerical techniques for computing approximate Lyapunov exponents, although they are not yet supported by a general error analysis. Here, a rigorous convergence theory is developed for both the discrete and continuous QR algorithm applied to a constant coefficient linear system with real distinct eigenvalues. For the discrete QR algorithm, the problem essentially reduces to one of linear algebra for which the timestepping and linear algebra errors uncouple and precise convergence rates are obtained. For the continuous QR algorithm, the stability, rather than the local accuracy, of the timestepping algorithm is relevant, and hence the overall convergence rate is independent of the stepsize. In this case it is vital to use a timestepping method that preserves orthogonality in the ODE system. We give numerical results to illustrate the analysis. Further numerical experiments and a heuristic argument suggest that the convergence properties carry through to the case of complex conjugate eigenvalue pairs.

Key words. dynamics, eigenvalues, orthogonal iteration, timestepping.

AMS subject classifications. 65L05, 65F15.

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[†]Department of Mathematics, University of Strathclyde, Glasgow, G1 1XH, Scotland. Email: ta.emcd@maths.strath.ac.uk.

[‡]Department of Mathematics, University of Strathclyde, Glasgow, G1 1XH, Scotland. Email: djh@maths.strath.ac.uk. This research was supported by the Engineering and Physical Sciences Research Council of the UK under grant GR/M42206.