Electronic Transactions on Numerical Analysis.
Volume 12, pp. 205-215, 2001.

ISSN 1068-9613.

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CHEBYSHEV APPROXIMATION VIA POLYNOMIAL MAPPINGS AND THE CONVERGENCE BEHAVIOUR OF KRYLOV SUBSPACE METHODS *

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Abstract. Let φ_m be a polynomial satisfying some mild conditions. Given a set $R \subset \mathbb{C}$, a continuous function f on R and its best approximation p_{n-1}^* from Π_{n-1} with respect to the maximum norm, we show that $p_{n-1}^* \circ \varphi_m$ is a best approximation to $f \circ \varphi_m$ on the inverse polynomial image S of R, i.e. $\varphi_m(S) = R$, where the extremal signature is given explicitly. A similar result is presented for constrained Chebyshev polynomial approximation. Finally, we apply the obtained results to the computation of the convergence rate of Krylov subspace methods when applied to a preconditioned linear system. We investigate pairs of preconditioners where the eigenvalues are contained in sets S and R, respectively, which are related by $\varphi_m(S) = R$.

Key words. Chebyshev polynomial, optimal polynomial, extremal signature, Krylov subspace method, convergence rate.

AMS subject classifications. 41A10, 30E10, 65F10.

*Received May 18, 1999. Accepted for publication June 22, 2001. Recommended by R. Freund.

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