GERŠGORIN-TYPE EIGENVALUE INCLUSION THEOREMS AND THEIR SHARPNESS[*]

RICHARD S. VARGA[†]

Dedicated to Professor John Todd, on the occasion of his 90th birthday, May 16, 2001.

Abstract. Here, we investigate the relationships between $\mathcal{G}(A)$, the union of Geršgorin disks, $\mathcal{K}(A)$, the union of Brauer ovals of Cassini, and $\mathcal{B}(A)$, the union of Brualdi lemniscate sets, for eigenvalue inclusions of an $n \times n$ complex matrix A. If $\sigma(A)$ denotes the spectrum of A, we show here that

$$\sigma(A) \subseteq \mathcal{B}(A) \subseteq \mathcal{K}(A) \subseteq \mathcal{G}(A)$$

is valid for any weakly irreducible $n \times n$ complex matrix A with $n \geq 2$. Further, it is evident that $\mathcal{B}(A)$ can contain the spectra of related $n \times n$ matrices. We show here that the spectra of these related matrices can fill out $\mathcal{B}(A)$. Finally, if $\mathcal{G}^{\mathcal{R}}(A)$ denotes the minimal Geršgorin set for A, we show that

$$\mathcal{G}^{\mathcal{R}}(A) \subseteq \mathcal{B}(A).$$

Key words. Geršgorin disks, Brauer ovals of Cassini, Brualdi lemniscate sets, minimal Geršgorin sets.

AMS subject classification. 15A18.