Electronic Transactions on Numerical Analysis
Volume 11, 2000

Contents

Abstract.
In this paper, we study some Schwarz methods of Neumann–Neumann type for some vector field problems, discretized with the lowest order Raviart–Thomas and Nedelec finite elements. We consider a hybrid Schwarz preconditioner consisting of a coarse component, which involves the solution of the original problem on a coarse mesh, and local ones, which involve the solution of Neumann problems on the elements of the coarse triangulation, also called substructures. We show that the condition number of the corresponding method is independent of the number of substructures and grows logarithmically with the number of unknowns associated with an individual substructure. It is also independent of the jumps of both the coefficients of the original problem. The numerical results presented validate our theoretical bound.

Key Words.
edge elements, Raviart–Thomas elements, domain decomposition, iterative substructuring, preconditioners, heterogeneous coefficients.

AMS(MOS) Subject Classifications.
65F10, 65N22, 65N30, 65N55.

Files.
vol.11.2000/pp1-24.dir/pp1-24.ps; vol.11.2000/pp1-24.dir/pp1-24.pdf;

Forward References.

Abstract.
The state-feedback pole (or eigenvalue) assignment problem is a fundamental problem in control system design. The term *eigenstructure* denotes the specification of eigenvalues *and* eigenvectors (or certain properties of the latter). Normally, the eigenvectors are calculated as an intermediate solution. In assignment for multi-input systems, the solution (the feedback matrix)is not unique. However, the solution is unique if the eigenvectors are set. Perturbation bounds are given for multi-input eigenstructure assignment of eigenvalues and eigenvectors occurring in complex-conjugate pairs. Numerical results which support the analysis are also provided.

Key Words.
controllable system, state feedback, eigenstructure assignment, multi-input pole assignment, perturbation analysis.

AMS(MOS) Subject Classifications.
15A18, 65F15, 65F35, 93B55.

Files.
vol.11.2000/pp25-42.dir/pp25-42.ps;
vol.11.2000/pp25-42.dir/pp25-42.pdf;

Forward References.

43 A multigrid method for saddle point problems arising from mortar finite element discretizations . *Barbara I. Wohlmuth* .

Abstract.
A multigrid algorithm for saddle point problems arising from mortar finite element discretizations is analyzed. Here, we do not require that the constraints at the interface are satisfied in each smoothing step, but we work on the squared system. Using mesh dependent norms for the Lagrange multipliers, suitable approximation and smoothing properties are established. A convergence rate independent of the meshsize is obtained for the $\mathcal{W}$–cycle.

Key Words.
mortar finite elements, saddle point problems, multigrid methods.

AMS(MOS) Subject Classifications.
65N22, 65N30, 65N55.

Files.
vol.11.2000/pp43-54.dir/pp43-54.ps;
vol.11.2000/pp43-54.dir/pp43-54.pdf;

Forward References.

55 High-order finite difference schemes and Toeplitz based preconditioners for elliptic problems. *Stefano Serra Capizzano and Cristina Tablino Possio.*

Abstract.
In this paper we are concerned with the spectral analysis of the sequence of preconditioned matrices

$$\{P_n^{-1}(a,m,k)A_n(a,m,k)\}_n,$$

where $A_n(a,m,k)$ is the $n \times n$ symmetric matrix coming from a high–order Finite Difference discretization of the problem

$$\begin{cases} (-)^k \left(\dfrac{d^k}{dx^k}\left(a(x)\dfrac{d^k}{dx^k}u(x)\right)\right) = f(x) \quad \text{on } \Omega = (0,1), \\ \left(\dfrac{d^s}{dx^s}u(x)\right)_{|\partial\Omega} = 0 \quad s = 0, \dots, k-1. \end{cases}$$

The coefficient function $a(x)$ is assumed to be positive or with a finite number of zeros. The matrix $P_n(a,m,k)$ is a Toeplitz based preconditioner constructed as $D_n^{1/2}(a,m,k)A_n(1,m,k)D_n^{1/2}(a,m,k)$, where $D_n(a,m,k)$ is the suitably scaled diagonal part of $A_n(a,m,k)$. The main result is the proof of the asymptotic clustering around unity of the eigenvalues of the preconditioned matrices. In addition, the "strength" of the cluster shows some interesting dependencies on the order k,

on the regularity features of $a(x)$ and on the presence of the zeros of $a(x)$. The multidimensional case is analyzed in depth in a twin paper [38].

Key Words.
finite differences, Toeplitz and Vandermonde matrices, clustering and preconditioning, ergodic theorems, spectral distribution.

AMS(MOS) Subject Classifications.
65N22, 65F10, 15A12.

Files.
vol.11.2000/pp55-84.dir/pp55-84.ps;
vol.11.2000/pp55-84.dir/pp55-84.pdf;

Forward References.

85 Cholesky-like factorizations of skew-symmetric matrices . *Peter Benner, Ralph Byers, Heike Fassbender, Volker Mehrmann, David Watkins.*

Abstract.
Every real skew-symmetric matrix B admits Cholesky-like factorizations $B = R^T JR$, where $J = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}$. This paper presents a backward-stable $\mathcal{O}(n^3)$ process for computing such a decomposition, in which R is a permuted triangular matrix. Decompositions of this type are a key ingredient of algorithms for solving eigenvalue problems with Hamiltonian structure.

Key Words.
skew-symmetric matrices, matrix factorizations, Hamiltonian eigenproblems, complete pivoting.

AMS(MOS) Subject Classifications.
15A23, 65F05.

Files.
vol.11.2000/pp85-93.dir/pp85-93.ps;
vol.11.2000/pp85-93.dir/pp85-93.pdf;

Forward References.

94 Numerical analysis of the radiosity equation using the collocation method. *Kendall Atkinson, David Da-Kwun Chien and Jaehoon Seol.*

Abstract.
The collocation method for solving the occluded radiosity equation is examined, theoretically and empirically. Theoretical results are examined, including questions of superconvergence of the collocation solution. The use of "discontinuity meshing" is examined for both piecewise constant and piecewise linear collocation. Also, numerical integration of the collocation integrals is examined, and a near-analytic evaluation method is given.

Key Words.
radiosity equation, integral equation, numerical integration, collocation methods, discontinuity meshing.

AMS(MOS) Subject Classifications.
65R20.

Files.
vol.11.2000/pp94-120.dir/pp94-120.ps;
vol.11.2000/pp94-120.dir/pp94-120.pdf;

Forward References.

121 Approximation of hypergeometric functions with matricial argument through their development in series of zonal polynomials. *R. Gutiérrez, J. Rodriguez and A. J. Sáez.*

Abstract.
Hypergeometric functions with matricial argument are being used in several fields of mathematics. This article tries to obtain a good approximation of this family of functions, since there are no general expressions for them, calculating zonal polynomials of high degrees and developing the functions in a truncated series.

Key Words.
hypergeometric functions with matricial argument, zonal polynomials.

AMS(MOS) Subject Classifications.
33C20, 33C99, 12Y05.

Files.
vol.11.2000/pp121-130.dir/pp121-130.ps;
vol.11.2000/pp121-130.dir/pp121-130.pdf;

Forward References.

131 Continuous Θ-methods for the stochastic pantograph equation. *Christopher T.H. Baker and Evelyn Buckwar .*

Abstract.
We consider a stochastic version of the pantograph equation:

$$dX(t) = \{aX(t) + bX(qt)\}\, dt + \{\sigma_1 + \sigma_2 X(t) + \sigma_3 X(qt)\}\, dW(t),$$
$$X(0) = X_0,$$

for $t \in [0, T]$, a given Wiener process W and $0 < q < 1$. This is an example of an Itô stochastic delay differential equation with unbounded memory. We give the necessary analytical theory for existence and uniqueness of a strong solution of the above equation, and of strong approximations to the solution obtained by a continuous extension of the Θ-Euler scheme ($\Theta \in [0, 1]$). We establish $\mathcal{O}(\sqrt{h})$ mean-square convergence of approximations obtained using a bounded mesh of uniform step h, rising in the case of additive noise to $\mathcal{O}(h)$. Illustrative numerical examples are provided.

Key Words.
stochastic delay differential equation, continuous Θ-method, mean-square convergence.

AMS(MOS) Subject Classifications.
65C30, 65Q05.

Files.
vol.11.2000/pp131-151.dir/pp131-151.ps;
vol.11.2000/pp131-151.dir/pp131-151.pdf;

Forward References.