Electronic Transactions on Numerical Analysis.
Volume 10, 2000, pp. 92-114.

ISSN 1068-9613.

ETNA
Kent State University
etna@mcs.kent.edu

BEHAVIOR OF PLANE RELAXATION METHODS AS MULTIGRID SMOOTHERS *

IGNACIO M. LLORENTE† AND N. DUANE MELSON‡

Abstract. This paper contains the first published numerical results and analysis of the behavior of alternating plane relaxation methods as multigrid smoothers for cell-centered grids. The results are very satisfactory: plane smoothers work well in general and their performance improves considerably for strong anisotropies in the right direction because they effectively become exact solvers. In fact, the convergence rate decreases (improves) linearly with increasing anisotropy strength. The methods compared are plane Jacobi with damping, plane Jacobi with partial damping, plane Gauss-Seidel, plane zebra Gauss-Seidel, and line Gauss-Seidel. Based on numerical experiments and local mode analysis, the smoothing factor and cost per cycle of the different methods in the presence of strong anisotropies for Dirichlet boundary conditions are compared. A four-color Gauss-Seidel method is found to have the best numerical and architectural properties of the methods considered in the present work. Although alternating direction plane relaxation schemes are simpler and more robust than other approaches, they are not currently used in industrial and production codes because they require the solution of a two-dimensional problem for each plane in each direction. We verify the theoretical predictions of Thole and Trottenberg that an exact solution of each plane is not necessary; in fact, a single two-dimensional multigrid cycle gives the same result as an exact solution, in much less execution time. As a result, alternating-plane smoothers are found to be highly efficient multigrid smoothers for anisotropic elliptic problems.

Key words. multigrid methods, anisotropic discrete operators, plane implicit methods, robust multigrid.

AMS subject classifications. 65M55.

---

* Received May 25, 1999. Accepted for publication February 1, 2000. Recommended by S. McCormick. This research was supported by the National Aeronautics and Space Administration under NASA Contract No. NAS1-97046 while the first author was in residence at the Institute for Computer Applications in Science and Engineering (ICASE), NASA Langley Research Center, Hampton, VA 23681-2199. The first author was supported by the *Dirección General de Enseñanza Superior* of the Spanish Government.

†Departamento de Arquitectura de Computadores y Automática, Universidad Complutense, 28040 Madrid, Spain (`llorente@dacya.ucm.es`).

‡NASA Langley Research Center, Hampton, VA 23681-2199 (`n.d.melson@larc.nasa.gov`).