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A NEW LEHMER PAIR OF ZEROS AND A NEW LOWER BOUND FOR THE DE BRUIJN-NEWMAN CONSTANT Λ*

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Dedicated to Wilhelm Niethammer on the occasion of his 60th birthday.

Abstract. The de Bruijn-Newman constant Λ has been investigated extensively because the truth of the Riemann Hypothesis is equivalent to the assertion that $\Lambda \leq 0$. On the other hand, C. M. Newman conjectured that $\Lambda \geq 0$. This paper improves previous lower bounds by showing that

$$-5.895 \cdot 10^{-9} < \Lambda.$$

This is done with the help of a spectacularly close pair of consecutive zeros of the Riemann zeta function.

Key words. Lehmer pairs of zeros, de Bruijn-Newman constant, Riemann Hypothesis.

AMS subject classifications. 30D10, 30D15, 65E05.

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