Appendix to "On simultaneous arithmetic progressions on elliptic curves". Examples of curves with s.a.p. of length 6

Irene García–Selfa José M. Tornero

Please note that this stream was computer–generated taking as a unique choice the permutation. Due to this, some of the curves (for instance examples 009 and 010) appear more than once, as they have different s.a.p. Also one may find isomorphic curves (as 001 and 002). As we said in the paper, only 56 non–isomorphic curves can be found in this table, but we have preferred to leave as it came, as we feel it illustrated better the phenomenon. The entries of the table after each equation are:

1) Permutation (σ): noted as in the paper by $(\sigma(0)\sigma(1)...\sigma(5))$.

2) Numerical data (N.D.): The set $[a, d, b, d']$ which fits the equation (see section 2).

3) Rank (r): The rank of the curve, computed with APECS ($\leq$ means APECS failed to actually compute the rank, in which case the best upper bound given is shown). All curves have trivial torsion group.

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001 $Y^2 - 180YX + 8100X^2 - X^3 + 4892251392X - 134063884477440$
$\sigma = (321450),\ \text{N.D.} = [66432, -20304, 13044672, -5725728],\ r = 5$

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002 $Y^2 + 180YX + 8100X^2 - X^3 + 4892251392X - 134063884477440$
$\sigma = (054123),\ \text{N.D.} = [-35088, 20304, -13044672, 5725728],\ r = 5$

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003 $Y^2 + 20YX + 100X^2 - X^3 + 36478512X - 82321246080$
$\sigma = (423150),\ \text{N.D.} = [5724, -1584, -367704, 139392],\ r = 5$

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004 $Y^2 - 60YX + 900X^2 - X^3 + 86832X - 8864640$

$\sigma = (534201),\ \text{N.D.} = [-324, 144, 9288, -3456],\ r = 2$

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005 $Y^2 - 20YX + 100X^2 - X^3 + 466992X - 549797760$
$\sigma = (045312),\ \text{N.D.} = [1308, -432, -33576, 3456],\ r = 3$

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006 $Y^2 - 2YX + X^2 - X^3 + 238707X - 41709006$
$\sigma = (150423),\ \text{N.D.} = [-513, 180, 8487, -3600],\ r = 3$

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007 $Y^2 - 180YX + 8100X^2 - X^3 + 51432192X - 368371860480$
$\sigma = (034521),\ \text{N.D.} = [-9312, 4752, -638496, 313632],\ r = 4$

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008 $Y^2 - 612YX + 93636X^2 - X^3 + 26962612992X - 1882111797863424$
$\sigma = (145032),\ \text{N.D.} = [-164832, 66960, -19904832, 12454560],\ r = 5$

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009 $Y^2 + 396YX + 39204X^2 - X^3 + 388106595072X - 88686989876929536$
$\sigma = (250143),\ \text{N.D.} = [-715488, 235440, 459510624, -153977760],\ r = 5$

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010 $Y^2 - 576YX + 82944X^2 - X^3 + 271059091200X - 49072046238950400$
$\sigma = (134502),\ \text{N.D.} = [734160, -265680, -415035360, 130714560],\ r = 5$

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011 $Y^2 - 576YX + 82944X^2 - X^3 + 271059091200X - 49072046238950400$
$\sigma = (350124),\ \text{N.D.} = [-594240, 265680, 238537440, -130714560],\ r = 5$

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012 $Y^2 + 396YX + 39204X^2 - X^3 + 388106595072X - 88686989876929536$
$\sigma = (305412),\ \text{N.D.} = [-715488, 235440, -310378176, 153977760],\ r = 5$

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013 $Y^2 - 612YX + 93636X^2 - X^3 + 26962612992X - 1882111797863424$
$\sigma = (410523),\ \text{N.D.} = [-164832, 66960, 42367968, -12454560],\ r = 5$

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014 $Y^2 - 180YX + 8100X^2 - X^3 + 51432192X - 368371860480$
$\sigma = (521034),\ \text{N.D.} = [-9312, 4752, 929664, -313632],\ r = 4$

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015 $Y^2 - 300YX + 22500X^2 - X^3 + 894074112X - 19561912750080$
$\sigma = (230145),\ \text{N.D.} = [-30384, 11808, -6045408, 2904768],\ r = 6$

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016 $Y^2 - 60YX + 900X^2 - X^3 + 48045312X - 189087436800$
$\sigma = (341250),\ \text{N.D.} = [-7344, 3744, -714528, 292032],\ r = 5$

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017 $Y^2 - 60YX + 900X^2 - X^3 + 48045312X - 189087436800$
$\sigma = (503412),\ \text{N.D.} = [11376, -3744, 745632, -292032],\ r = 5$

018	$Y^2 + 300YX + 22500X^2 - X^3 + 894074112X - 19561912750080$ $\sigma = (014523),\ \text{N.D.} = [28656, -11808, -8478432, 2904768],\ r = 6$
019	$Y^2 - 1800YX + 810000X^2 - X^3 + 7410269014272X - 8108956611489899520$ $\sigma = (302145),\ \text{N.D.} = [2763696, -577584, 3502719072, -1323822528],\ r \leq 6$
020	$Y^2 + 2880YX + 2073600X^2 - X^3 + 403246536682752X - 3207719147336296058880$ $\sigma = (413250),\ \text{N.D.} = [-829968, 5312736, 62302243392, -30155089536],\ r \leq 6$
021	$Y^2 + 4680YX + 5475600X^2 - X^3 + 3994541989632X - 6207305561351930880$ $\sigma = (524301),\ \text{N.D.} = [-2327712, 728784, -3390388704, 2107643328],\ r \leq 7$
022	$Y^2 - 2880YX + 2073600X^2 - X^3 + 8792422324992X - 14409886139859502080$ $\sigma = (035412),\ \text{N.D.} = [7563792, -2228688, -8618074272, 1791865152],\ r \leq 11$
023	$Y^2 + 7632YX + 14561856X^2 - X^3 + 1208964238739712X - 23944945010158503235584$ $\sigma = (140523),\ \text{N.D.} = [-42040752, 12290400, 265952884032, -83918851200],\ r \leq 7$
024	$Y^2 - 6912YX + 11943936X^2 - X^3 + 214026269587200X - 1973672371225801958400$ $\sigma = (250134),\ \text{N.D.} = [-9752640, 7294320, 76016037600, -26872274880],\ r \leq 8$
025	$Y^2 + 120YX + 3600X^2 - X^3 + 81948672X - 327577374720$ $\sigma = (532140),\ \text{N.D.} = [8064, -3168, 1169856, -418176],\ r = 4$
026	$Y^2 + 960YX + 230400X^2 - X^3 + 92277352334592X - 342160070800370356224$ $\sigma = (043251),\ \text{N.D.} = [11544048, -3013920, -34095021120, 10813944960],\ r = 4$
027	$Y^2 + 2616YX + 1710864X^2 - X^3 + 3327107104512X - 3360005144504534016$ $\sigma = (154302),\ \text{N.D.} = [-515808, 403920, -2454087456, 906396480],\ r \leq 6$
028	$Y^2 - 3072YX + 2359296X^2 - X^3 + 2328753038487552X - 49786655684722942869504$ $\sigma = (205413),\ \text{N.D.} = [-55130112, 19313280, 240362650368, -111012733440],\ r \leq 6$
029	$Y^2 - 4944YX + 6110784X^2 - X^3 + 14552162687232X - 21391150302252205056$ $\sigma = (310524),\ \text{N.D.} = [1164672, 340560, 5468006304, -44953920],\ r \leq 7$
030	$Y^2 - 960YX + 230400X^2 - X^3 + 19321742592X - 1033886822584320$ $\sigma = (421035),\ \text{N.D.} = [60912, 6048, 38636352, -72576],\ r = 5$

031 $Y^2 + 40YX + 400X^2 - X^3 + 8854272X - 8259978240$
$\sigma = (153420),\ \text{N.D.} = [-2448, 1584, 242208, -69696],\ r = 4$

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032 $Y^2 - 64YX + 1024X^2 - X^3 + 1389312X - 394896384$
$\sigma = (204531),\ \text{N.D.} = [-1152, 720, -44064, 14400],\ r = 3$

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033 $Y^2 - 64YX + 1024X^2 - X^3 + 1389312X - 394896384$
$\sigma = (420153),\ \text{N.D.} = [2448, -720, 27936, -14400],\ r = 3$

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034 $Y^2 - 40YX + 400X^2 - X^3 + 8854272X - 8259978240$
$\sigma = (531204),\ \text{N.D.} = [5472, -1584, 106272, -69696],\ r = 4$

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035 $Y^2 - 1080YX + 291600X^2 - X^3 + 4379037151491072X - 136063103467431710822400$
$\sigma = (240315),\ \text{N.D.} = [-77988192, 33054048, -438167226816, 226486336896],\ r \leq 8$

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036 $Y^2 - 720YX + 129600X^2 - X^3 + 9969629720832X - 13778174775900128256$
$\sigma = (351420),\ \text{N.D.} = [-3640128, 1648080, -6422790240, 2155688640],\ r = 1$

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037 $Y^2 + 5544YX + 7683984X^2 - X^3 + 422337203926272X - 3401406251614088487936$
$\sigma = (402531),\ \text{N.D.} = [-18956688, 10568880, 102961496736, -29550588480],\ r \leq 7$

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038 $Y^2 + 1080YX + 291600X^2 - X^3 + 4379037151491072X - 136063103467431710822400$
$\sigma = (513042),\ \text{N.D.} = [87282048, -33054048, 438167226816, -226486336896],\ r \leq 8$

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039 $Y^2 - 720YX + 129600X^2 - X^3 + 9969629720832X - 13778174775900128256$
$\sigma = (024153),\ \text{N.D.} = [4600272, -1648080, -6422790240, 2155688640],\ r = 1$

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040 $Y^2 - 5544YX + 7683984X^2 - X^3 + 422337203926272X - 3401406251614088487936$
$\sigma = (135204),\ \text{N.D.} = [33887712, -10568880, -102961496736, 29550588480],\ r \leq 7$

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041 $Y^2 + 20YX + 100X^2 - X^3 + 466992X - 549797760$
$\sigma = (342015),\ \text{N.D.} = [-852, 432, 16296, 3456],\ r = 3$

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042 $Y^2 - 60YX + 900X^2 - X^3 + 86832X - 8864640$
$\sigma = (453120),\ \text{N.D.} = [396, -144, -7992, 3456],\ r = 2$

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043 $Y^2 - 20YX + 100X^2 - X^3 + 36478512X - 82321246080$
$\sigma = (504231),\ \text{N.D.} = [-2196, 1584, -329256, 139392],\ r = 5$

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044 $Y^2 - 4YX + 4X^2 - X^3 + 3819312X - 2669376384$

$\sigma = (231504),\ \text{N.D.} = [1548, -720, -76104, 28800],\ r = 3$

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045 $Y^2 - 2880YX + 2073600X^2 - X^3 + 110993807215872X - 445121432212494274560$
$\sigma = (402315),\ \text{N.D.} = [14071152, -3627936, 35652163392, -14061879936],\ r \leq 7$

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046 $Y^2 + 360YX + 32400X^2 - X^3 + 27909792000X - 1526822144640000$
$\sigma = (513420),\ \text{N.D.} = [-99600, 75600, 73720800, -22680000],\ r \leq 5$

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047 $Y^2 - 7056YX + 12446784X^2 - X^3 + 129437671756032X - 543091962160151110656$
$\sigma = (135042),\ \text{N.D.} = [17527152, -4944240, -5119105824, 6467065920],\ r \leq 5$

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048 $Y^2 - 7488YX + 14017536X^2 - X^3 + 841545692773632X - 15278101189303210758144$
$\sigma = (240153),\ \text{N.D.} = [-34464912, 10864800, -202374030528, 65579932800],\ r \leq 8$

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049 $Y^2 - 6696YX + 11209104X^2 - X^3 + 1767944739520512X - 27067552170879621206016$
$\sigma = (351204),\ \text{N.D.} = [-46912128, 22001760, 291906110016, -122241778560],\ r \leq 7$

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050 $Y^2 - 90YX + 2025X^2 - X^3 + 3434834187X - 77407635019590$
$\sigma = (053421),\ \text{N.D.} = [39747, -2376, 3706047, -627264],\ r = 5$

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051 $Y^2 + 306YX + 23409X^2 - X^3 + 338359707X - 3186238861494$
$\sigma = (104532),\ \text{N.D.} = [-21237, 11880, 3783861, -1425600],\ r = 4$

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052 $Y^2 - 612YX + 93636X^2 - X^3 + 5413755312X - 203919287135616$
$\sigma = (320154),\ \text{N.D.} = [152652, -47520, 26753112, -11404800],\ r = 4$

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053 $Y^2 - 90YX + 2025X^2 - X^3 + 3434834187X - 77407635019590$
$\sigma = (431205),\ \text{N.D.} = [27867, 2376, 569727, 627264],\ r = 5$

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054 $Y^2 + 396YX + 39204X^2 - X^3 + 388106595072X - 88686989876929536$
$\sigma = (214503),\ \text{N.D.} = [461712, -235440, -310378176, 153977760],\ r = 5$

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055 $Y^2 - 612YX + 93636X^2 - X^3 + 26962612992X - 1882111797863424$
$\sigma = (325014),\ \text{N.D.} = [169968, -66960, 42367968, -12454560],\ r = 5$

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056 $Y^2 - 180YX + 8100X^2 - X^3 + 51432192X - 368371860480$
$\sigma = (430125),\ \text{N.D.} = [14448, -4752, 929664, -313632],\ r = 4$

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057 $Y^2 - 48YX + 576X^2 - X^3 + 9262512X - 8383430016$
$\sigma = (250341),\ \text{N.D.} = [-3348, 1440, -123552, 43200],\ r = 2$

058	$Y^2 - 24YX + 144X^2 - X^3 + 578907X - 130991094$ $\sigma = (412503),\ \text{N.D.} = [963, -360, 11556, -5400],\ r = 2$
059	$Y^2 - 60YX + 900X^2 - X^3 + 767232X - 253808640$ $\sigma = (523014),\ \text{N.D.} = [1008, -288, 16416, -1728],\ r = 3$
060	$Y^2 - 60YX + 900X^2 - X^3 + 767232X - 253808640$ $\sigma = (145230),\ \text{N.D.} = [-432, 288, 7776, 1728],\ r = 3$
061	$Y^2 - 5184YX + 6718464X^2 - X^3 + 142783615579392X - 708085096419827994624$ $\sigma = (352041),\ \text{N.D.} = [13189008, -5387040, -40963299264, 13898563200],\ r = 4$
062	$Y^2 - 2142YX + 1147041X^2 - X^3 + 15808640228307X - 39293749322977053294$ $\sigma = (403152),\ \text{N.D.} = [5885823, -1961820, 10341141993, -4072738320],\ r \leq 7$
063	$Y^2 - 612YX + 93636X^2 - X^3 + 52888734000X - 42522157354464000$ $\sigma = (514203),\ \text{N.D.} = [-243780, 110160, 98574840, -44945280],\ r \leq 5$
064	$Y^2 - 396YX + 39204X^2 - X^3 + 16655166000X - 690091901769600$ $\sigma = (025314),\ \text{N.D.} = [250140, -71280, -60813720, 18817920],\ r = 4$
065	$Y^2 - 1710YX + 731025X^2 - X^3 + 1041760268307X - 785868626594613294$ $\sigma = (130425),\ \text{N.D.} = [-1270977, 427140, -1427543055, 579201840],\ r \leq 5$
066	$Y^2 - 3240YX + 2624400X^2 - X^3 + 2359518446592X - 1392039372309073920$ $\sigma = (241530),\ \text{N.D.} = [502176, 175392, 1361800512, 14732928],\ r = 7$
067	$Y^2 - 60YX + 900X^2 - X^3 + 48045312X - 189087436800$ $\sigma = (503412),\ \text{N.D.} = [11376, -3744, 745632, -292032],\ r = 5$
068	$Y^2 + 300YX + 22500X^2 - X^3 + 894074112X - 19561912750080$ $\sigma = (014523),\ \text{N.D.} = [28656, -11808, -8478432, 2904768],\ r = 6$
069	$Y^2 - 300YX + 22500X^2 - X^3 + 894074112X - 19561912750080$ $\sigma = (230145),\ \text{N.D.} = [-30384, 11808, -6045408, 2904768],\ r = 6$
070	$Y^2 - 60YX + 900X^2 - X^3 + 48045312X - 189087436800$ $\sigma = (341250),\ \text{N.D.} = [-7344, 3744, -714528, 292032],\ r = 5$

071 $Y^2 - 960YX + 230400X^2 - X^3 + 13201887036672X - 17372103328571019264$
$\sigma = (502341),\ \text{N.D.} = [5909328, -1703520, 8039027520, -3454738560],\ r = 6$

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072 $Y^2 + 600YX + 90000X^2 - X^3 + 4185188352X - 119899753943040$
$\sigma = (013452),\ \text{N.D.} = [52704, -12384, -22572864, 6390144],\ r \leq 7$

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073 $Y^2 + 768YX + 147456X^2 - X^3 + 28465015272192X - 52273748736104702976$
$\sigma = (124503),\ \text{N.D.} = [4297392, -2075760, -8562479328, 3860913600],\ r \leq 6$

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074 $Y^2 - 4368YX + 4769856X^2 - X^3 + 102164509274112X - 521183246173070966784$
$\sigma = (235014),\ \text{N.D.} = [7567488, -3850560, 20709101952, -8825483520],\ r \leq 6$

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075 $Y^2 - 192YX + 9216X^2 - X^3 + 1067634432X - 20407326188544$
$\sigma = (340125),\ \text{N.D.} = [-32832, 12240, -6162912, 2496960],\ r = 4$

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076 $Y^2 + 120YX + 3600X^2 - X^3 + 210573084672X - 33546735732363264$
$\sigma = (451230),\ \text{N.D.} = [-495072, 211680, -339888960, 124467840],\ r = 6$

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077 $Y^2 - 180YX + 8100X^2 - X^3 + 51432192X - 368371860480$
$\sigma = (125430),\ \text{N.D.} = [14448, -4752, -638496, 313632],\ r = 4$

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078 $Y^2 - 612YX + 93636X^2 - X^3 + 26962612992X - 1882111797863424$
$\sigma = (230541),\ \text{N.D.} = [169968, -66960, -19904832, 12454560],\ r = 5$

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079 $Y^2 + 396YX + 39204X^2 - X^3 + 388106595072X - 88686989876929536$
$\sigma = (341052),\ \text{N.D.} = [461712, -235440, 459510624, -153977760],\ r = 5$

---

080 $Y^2 - 60YX + 900X^2 - X^3 + 48045312X - 189087436800$
$\sigma = (214305),\ \text{N.D.} = [-7344, 3744, 745632, -292032],\ r = 5$

---

081 $Y^2 - 300YX + 22500X^2 - X^3 + 894074112X - 19561912750080$
$\sigma = (325410),\ \text{N.D.} = [-30384, 11808, 8478432, -2904768],\ r = 6$

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082 $Y^2 + 300YX + 22500X^2 - X^3 + 894074112X - 19561912750080$
$\sigma = (541032),\ \text{N.D.} = [28656, -11808, 6045408, -2904768],\ r = 6$

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083 $Y^2 - 60YX + 900X^2 - X^3 + 48045312X - 189087436800$
$\sigma = (052143),\ \text{N.D.} = [11376, -3744, -714528, 292032],\ r = 5$

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084 $Y^2 - 3240YX + 2624400X^2 - X^3 + 2359518446592X - 1392039372309073920$

$\sigma = (314025),\ \text{N.D.} = [502176, 175392, 1435465152, -14732928],\ r = 7$

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085 $Y^2 - 1710YX + 731025X^2 - X^3 + 1041760268307X - 785868626594613294$
$\sigma = (425130),\ \text{N.D.} = [-1270977, 427140, 1468466145, -579201840],\ r \leq 5$

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086 $Y^2 - 396YX + 39204X^2 - X^3 + 16655166000X - 690091901769600$
$\sigma = (530241),\ \text{N.D.} = [250140, -71280, 33275880, -18817920],\ r = 4$

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087 $Y^2 - 612YX + 93636X^2 - X^3 + 52888734000X - 4252215735446400$
$\sigma = (041352),\ \text{N.D.} = [-243780, 110160, -126151560, 44945280],\ r \leq 5$

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088 $Y^2 + 4284YX + 4588164X^2 - X^3 + 252938243652912X - 2514799956670531410816$
$\sigma = (152403),\ \text{N.D.} = [23543292, -7847280, 80180396856, -32581906560],\ r \leq 7$

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089 $Y^2 - 5184YX + 6718464X^2 - X^3 + 142783615579392X - 708085096419827994624$
$\sigma = (203514),\ \text{N.D.} = [13189008, -5387040, 28529516736, -13898563200],\ r = 4$

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090 $Y^2 - 60YX + 900X^2 - X^3 + 767232X - 253808640$
$\sigma = (410325),\ \text{N.D.} = [-432, 288, 16416, -1728],\ r = 3$

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091 $Y^2 - 60YX + 900X^2 - X^3 + 767232X - 253808640$
$\sigma = (032541),\ \text{N.D.} = [1008, -288, 7776, 1728],\ r = 3$

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092 $Y^2 - 24YX + 144X^2 - X^3 + 578907X - 130991094$
$\sigma = (143052),\ \text{N.D.} = [963, -360, -15444, 5400],\ r = 2$

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093 $Y^2 - 48YX + 576X^2 - X^3 + 9262512X - 8383430016$
$\sigma = (305214),\ \text{N.D.} = [-3348, 1440, 92448, -43200],\ r = 2$

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094 $Y^2 + 120YX + 3600X^2 - X^3 + 81948672X - 327577374720$
$\sigma = (514320),\ \text{N.D.} = [-7776, 3168, -921024, 418176],\ r = 4$

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095 $Y^2 - 960YX + 230400X^2 - X^3 + 19321742592X - 1033886822584320$
$\sigma = (025431),\ \text{N.D.} = [91152, -6048, 38273472, 72576],\ r = 5$

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096 $Y^2 - 4944YX + 6110784X^2 - X^3 + 14552162687232X - 21391150302252205056$
$\sigma = (130542),\ \text{N.D.} = [2867472, -340560, 5243236704, 44953920],\ r \leq 7$

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097 $Y^2 - 3072YX + 2359296X^2 - X^3 + 2328753038487552X - 49786655684722942869504$
$\sigma = (241053),\ \text{N.D.} = [41436288, -19313280, -314701016832, 111012733440],\ r \leq 6$

098	$Y^2 - 2616YX + 1710864X^2 - X^3 + 3327107104512X - 3360005144504534016$ $\sigma = (352104),\ \text{N.D.} = [1503792, -403920, -2077894944, 906396480],\ r \leq 6$
099	$Y^2 - 960YX + 230400X^2 - X^3 + 92277352334592X - 342160070800370356224$ $\sigma = (403215),\ \text{N.D.} = [-3525552, 3013920, -19974703680, 10813944960],\ r \leq 4$
100	$Y^2 - 60YX + 900X^2 - X^3 + 767232X - 253808640$ $\sigma = (145230),\ \text{N.D.} = [-432, 288, 7776, 1728],\ r = 3$