Calculating Canonical Distinguished Involutions in the Affine Weyl Groups

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CONTENTS

Distinguished involutions in the affine Weyl groups, defined by G. Lusztig, play an essential role in the Kazhdan-Lusztig combinatorics of these groups. A distinguished involution is called canonical if it is the shortest element in its double coset with respect to the finite Weyl group. Each two-sided cell in the affine Weyl group contains precisely one canonical distinguished involution. We calculate the canonical distinguished involutions in the affine Weyl groups of rank ≤ 7. We also prove some partial results relating canonical distinguished involutions and Dynkin's diagrams of the nilpotent orbits in the Langlands dual group.

1. INTRODUCTION

It is well known that the Kazhdan-Lusztig combinatorics of the affine Hecke algebra is related to the geometry of the corresponding algebraic group (over the complex numbers) G in a fundamental way. The central result is the deep theorem of George Lusztig establishing a bijection between the set $\mathfrak{U}$ of unipotent classes in G and the set of *two-sided cells* in the corresponding affine Weyl group W_a, see [Lusztig 89]. Using this result, one defines a map from $\mathfrak{U}$ to the set X_+ of the dominant weights in the following way: let $\mathcal{O}$ be an unipotent orbit and let $\mathbf{c}_\mathcal{O}$ be the corresponding two-sided cell. George Lusztig and Nanhua Xi attached to $\mathbf{c}_\mathcal{O}$ a *canonical left cell* $C_\mathcal{O}$, see [Lusztig 88]. In turn, the left cell $C_\mathcal{O}$ contains a unique *distinguished involution* $d_\mathcal{O} \in W_a$, see [Lusztig 87], which is the shortest element in its double coset $W d_\mathcal{O} W$ with respect to the finite Weyl group $W \subset W_a$. It is well known that the set of double cosets $W \backslash W_a / W$ is bijective to the set X_+ since any such coset contains a unique translation by a dominant weight. Combining the maps above, we get a canonical map $\mathcal{L} : \mathfrak{U} \to X_+$. The explicit calculation of this map is equivalent to the determination of the distinguished involutions lying in the canonical cells (or, equivalently, those cells which are shortest in their

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left coset with respect to the finite Weyl group). We call such involutions *canonical distinguished involutions*. We believe that understanding these involutions is an important step towards the understanding of all distinguished involutions and cells in the affine Weyl group.

In [Ostrik 00], one of the authors suggested an algorithm for the calculation of $\mathcal{L}$, and now this algorithm is known to be correct thanks to the (unfortunately still undocumented) work of R. Bezrukavnikov. The aim of this paper is to present results of calculations made using this algorithm.

In Section 2 we present the necessary background. In Section 3, we present our main results: the calculation of the map $\mathcal{L}$ for the group GL_n and partial results for other groups. These results are apparently known to the experts but, to the best of our knowledge, they have never been published. In Section 4, we present tables giving the results of explicit calculation of the map $\mathcal{L}$ for groups of small rank. These tables should be considered as a main result of this work.

2. BACKGROUND

2.1 Notations

Let G be a semisimple algebraic group over the complex numbers. Let $\mathfrak{g}$ denote the Lie algebra of G and let $\mathcal{N} \subset \mathfrak{g}$ denote the nilpotent cone. As a $G-$variety, $\mathcal{N}$ is isomorphic to the subvariety of unipotent elements of G via the exponential map but it has one additional property—an obvious action of $\mathbb{C}^*$ by dilations commuting with the $G-$action. We will consider $\mathcal{N}$ a $G \times \mathbb{C}^*-$variety via the following action: $(g, z)n = z^{-2}Ad(g)n$ for $(g, z) \in G \times \mathbb{C}^*$ and $n \in \mathcal{N}$.

The variety $\mathcal{N}$ consists of finitely many $G-$orbits, [Collingwood, McGovern 93]. These orbits, called *nilpotent orbits*, are the main subject of our study. Any nilpotent orbit $\mathcal{O}$ is identified via its *Dynkin diagram* defined as follows: let $e \in \mathcal{O}$ be a representative. By the Jacobson–Morozov Theorem, it can be included in the sl_2-triple (e, f, h) (i.e., $[h, e] = 2e,\ [h, f] = -2f,\ [e, f] = h$). The semisimple element h is uniquely defined up to $G-$conjugacy by $\mathcal{O}$ and vice versa, [Collingwood, McGovern 93]. Let $\mathfrak{h} \subset \mathfrak{g}$ be a Cartan subalgebra, let $R \subset \mathfrak{h}^*$ be the root system, then choose a subset $R_+ \subset R$ of positive roots and let $\{\alpha_i, i \in I\}$ be the set of simple roots (I is the set of vertices of the Dynkin diagram of $\mathfrak{g}$). The element h is conjugate to a unique $h_0 \in \mathfrak{h}$ such that $\alpha_i(h_0)$ is positive for any $i \in I$ and, moreover, $\alpha_i(h_0) \in \{0, 1, 2\}$, see [Collingwood, McGovern 93]. Thus any nilpotent orbit can be identified by labeling the Dynkin diagram of $\mathfrak{g}$ by numbers $0, 1, 2$. This is called the (labeled) Dynkin diagram of $\mathfrak{g}$. We note that the Dynkin diagram h_0 is naturally an integral dominant *coweight* for group G.

Let X denote the weight lattice of G and let X_+ denote the set of dominant weights.

2.2 Equivariant K-Theory of the Nilpotent Cone

In this section, we review [Ostrik 00]. Let $K_{G\times\mathbb{C}^*}(\mathcal{N})$ denote the Grothendieck group of the category of $G \times \mathbb{C}^*-$equivariant coherent sheaves on $\mathcal{N}$. It has an obvious structure as a module over the representation ring $Rep(\mathbb{C}^*)$ of $\mathbb{C}^*$. Let v denote the tautological representation of $\mathbb{C}^*$. Then $Rep(\mathbb{C}^*) = \mathbb{Z}[v, v^{-1}]$ and the rule $v \mapsto v^{-1}$ defines an involution $\bar{:} Rep(\mathbb{C}^*) \to Rep(\mathbb{C}^*)$.

In [Ostrik 00] the following were constructed:

1. the basis $\{AJ(\lambda)\}$ of $K_{G\times\mathbb{C}^*}(\mathcal{N})$ over $Rep(\mathbb{C}^*) = \mathbb{Z}[v, v^{-1}]$ labeled by dominant weights $\lambda \in X_+$.

2. a $Rep(\mathbb{C}^*)-$antilinear involution $K_{G\times\mathbb{C}^*}(\mathcal{N}) \to K_{G\times\mathbb{C}^*}(\mathcal{N}), x \mapsto \bar{x}$.

Then, using usual Kazhdan-Lusztig machinery, the basis $\{C(\lambda)\}$ was defined. Thus $C(\lambda)$ is a unique self-dual element of the form $AJ(\lambda)+\sum_{\mu<\lambda} b_{\mu,\lambda}AJ(\mu)$ where $b_{\mu,\lambda} \in v^{-1}\mathbb{Z}[v^{-1}]$). The main conjecture is that for any $\lambda \in X_+$ the support of $C(\lambda)$ is the closure of a nilpotent orbit $\mathcal{O}_\lambda$ and that $C(\lambda)|_{\mathcal{O}_\lambda}$ represents, up to sign, the class of an irreducible $G-$equivariant bundle on $\mathcal{O}_\lambda$. In this way, we can recover Lusztig's bijection between dominant weights X_+ and pairs consisting of a nilpotent orbit and $G-$equivariant irreducible bundle on it (see [Lusztig 89], [Bezrukavnikov 00], [Bezrukavnikov 01], [Vogan 00] for various approaches to Lusztig's bijection). All these conjectures are now known to be true thanks to the work of R. Bezrukavnikov.

Now let $e^\lambda \in K_G(\mathcal{N})$ be the image of $AJ(\lambda)$ under the forgetting map $K_{G\times\mathbb{C}^*}(\mathcal{N}) \to K_G(\mathcal{N})$. By definition, e^λ can be constructed as follows: let $\mathcal{L}(\lambda)$ be the line bundle on G/B corresponding to the weight λ (we choose the notation in such a way that $H^0(G/B, \mathcal{L}(\lambda)) \neq 0$ for *dominant* λ); then $e^\lambda = [sp_*\pi^*\mathcal{L}(\lambda)]$ where $\pi : T^*G/B \to G/B$ is the natural projection and $sp : T^*G/B \to \mathcal{N}$ is the Springer resolution. (Here sp_* denotes the direct image in the derived category of coherent sheaves or, equivalently, $[sp_*\pi^*\mathcal{L}(\lambda)]$ is the alternating sum of the higher direct images $R^i sp_*$.) This definition makes sense for any (not necessarily dominant) weight λ and we know that $e^{w\lambda} = e^\lambda$ for any $w \in W, \lambda \in X$.

2.3 McGovern's Formula

We are especially interested in weights corresponding to the trivial bundles on nilpotent orbits under Lusztig's bijection. One of the conjectures in [Ostrik 00] states that these weights are exactly $\mathcal{L}(\mathcal{O})$ with $\mathcal{L}$ as defined in Section 1. Moreover, $C(\mathcal{L}(\mathcal{O}))$ represents a class $j_*(\mathbb{C}_\mathcal{O})$ where $j : \mathcal{O} \to \mathcal{N}$ is the natural inclusion and $\mathbb{C}_\mathcal{O}$ is the trivial $G-$equivariant bundle on $\mathcal{O}$. Again, this is known to be true thanks to the work of R. Bezrukavnikov.

There is a simple formula for $j_*(\mathbb{C}_\mathcal{O})$ due to W. McGovern: Let h be the Dynkin diagram of the orbit $\mathcal{O}$. Then h defines a grading of Lie algebra $\mathfrak{g} = \bigoplus_{i\in\mathbb{Z}} \mathfrak{g}_i$ where $\mathfrak{g}_i = \{x \in \mathfrak{g} | [h, x] = ix\}$. Furthermore, $\mathfrak{g}_{\geq 0} = \bigoplus_{i\geq 0} \mathfrak{g}_i$ is a parabolic subalgebra of $\mathfrak{g}$ and $\mathfrak{g}_{\geq 2} = \bigoplus_{i\geq 2} \mathfrak{g}_i$ is a module over $\mathfrak{g}_{\geq 0}$. Let $G_{\geq 0}$ be a parabolic subgroup with Lie algebra $\mathfrak{g}_{\geq 0}$ and let $M = G \times_{G_{\geq 0}} \mathfrak{g}_{\geq 2}$ be the homogeneous bundle on $G/G_{\geq 0}$ corresponding to the G_0-module $\mathfrak{g}_{\geq 2}$. There is a natural map $r : G \times_{G_{\geq 0}} \mathfrak{g}_{\geq 2} \to \mathfrak{g}$. The image of r is exactly $\bar{\mathcal{O}}$ and, moreover, this map is proper and generically one to one and therefore it is a resolution of singularities of $\bar{\mathcal{O}}$ [McGovern 89]. McGovern proved that $[j_*\mathbb{C}_\mathcal{O}] = [r_*\mathbb{C}_M]$.

Now let $R_{+,0}$ (resp. $R_{+,1}$) be the subset of all positive roots such that $\alpha(h) = 0$ (resp. $\alpha(h) = 1$). Then the Koszul complex gives the following formula:

$$[j_*\mathbb{C}_\mathcal{O}] = \prod_{\alpha\in R_{+,0}\cup R_{+,1}} (e^0 - e^\alpha) \qquad (*)$$

(see [McGovern 89] for details). This suggests the following algorithm for computing $\mathcal{L}(\mathcal{O})$:

1. Multiply the brackets on the right hand side of (*) using the usual rule, $e^\lambda e^\mu = e^{\lambda+\mu}$.
2. In the expression derived from Step 1, replace each e^λ by $e^{w\lambda}$ where $w \in W$ and $w\lambda$ is dominant. Then, make all possible cancellations. (Warning: Step 1 and Step 2 do not commute!)
3. In the expression from Step 2, find the leading term $\pm e^\lambda$ such that for any other term e^μ the inequality $\lambda > \mu$ holds (the existence of such λ is a consequence of the results in [Bezrukavnikov 01]). This λ is exactly $\mathcal{L}(\mathcal{O})$.

Unfortunately this algorithm is completely impractical for groups of large rank.

3. THEOREMS

The results of this section are probably well known to experts. Moreover, Theorem 3.5 was stated in [Ostrik 00] without proof.

3.1 Richardson Resolutions

It follows from the theorem of Hinich and Paniushev [Hinich 91], [Panyushev 91] on the rationality of singularities of normalizations of closures of nilpotent orbits that $j_*\mathbb{C}_\mathcal{O} = r_*\mathbb{C}_M$ where $r : M \to \bar{\mathcal{O}}$ is a resolution of singularities of $\bar{\mathcal{O}}$. One obtains McGovern's formula from this using the canonical resolution of a nilpotent orbit. Now, let P be a parabolic subgroup of G. The image of the moment map $m_P : T^*G/P \to \mathfrak{g}^* = \mathfrak{g}$ is the closure of a nilpotent orbit $\mathcal{O}(P)$ and by a well known theorem due to R. W. Richardson the map $m_P : T^*G/P \to \overline{\mathcal{O}(P)}$ is generically finite to one.

Let $G^\vee$ be a Langlands dual group of G. By definition there is a bijection between simple roots of G and $G^\vee$. In particular, we can attach a Levi subgroup $L_P^\vee \subset G^\vee$ to any parabolic subgroup $P \subset G$. Recall that any coweight for $G^\vee$ is by definition a weight for G.

Theorem 3.1. *Let $P \subset G$ be a parabolic subgroup such that the map $m_P : T^*G/P \to \overline{\mathcal{O}(P)}$ is birational. Then $\mathcal{L}(\mathcal{O}_P)$ is equal to the Dynkin diagram of the principal nilpotent element in $L_P^\vee$.*

Proof: From the above remarks, we know that $j_*\mathbb{C}_{\mathcal{O}_P} = m_*\mathbb{C}_{T^*G/P}$. Let $R_+(L) \subset R_+$ denote the subset of positive roots of the subgroup L. Again the Koszul complex gives a formula

$$m_*\mathbb{C}_{T^*G/P} = \prod_{\alpha\in R_+(L)} (1 - e^\alpha)$$

interpreted in the same way as McGovern's formula (Section 2.3). Let $\rho_L = \frac{1}{2}\sum_{\alpha\in R_+(L)} \alpha$ and let $W_L \subset W$ denote the Weyl group of L. The Weyl denominator formula gives

$$\prod_{\alpha\in R_+(L)} (1 - e^\alpha) = \sum_{w\in W_L} det(w)e^{\rho_L - w\rho_L}.$$

In particular we have a leading term $e^{2\rho_L}$ corresponding to the summand with $w = w_0(L)$—the element of largest length in W_L. It cannot cancel with anything else since the scalar product $\langle 2\rho_L, 2\rho_L\rangle$ is clearly bigger than any scalar product $\langle \rho_L - w\rho_L, \rho_L - w\rho_L\rangle$ for $w \neq w_0(L)$. For the same reason, this term is the unique candidate for the leading term since $\lambda > \mu$ for dominant λ and μ which implies $\langle\lambda, \lambda\rangle > \langle\mu, \mu\rangle$. Since we know that the leading term exists (this is Bezrukavnikov's result as we mentioned in Section 2.3) it should be equal to $e^{x\rho_L}$ where $x \in W$ is such that $x\rho_L$ is dominant.

Finally weight $2\rho_L$, considered as a coweight for $G^\vee$, is clearly the Dynkin diagram of the regular nilpotent element in $L^\vee$ since the Dynkin diagram of a regular nilpotent element is always the sum of positive coweights, see [Kostant 63]. □

3.2 Nilpotent Orbits

We will say that a nilpotent orbit is a *strongly Richardson orbit* if it admits a desingularization via the momentum map $m_P : T^*G/P \to \bar{\mathcal{O}}$. We have the following

Theorem 3.2. *Let $\mathcal{O} = \mathcal{O}_P$ be a strongly Richardson orbit. Suppose that the $G^\vee-$orbit $\mathcal{O}'$ of a regular nilpotent element of $L_P^\vee$ is also strongly Richardson in $G^\vee$. Then $\mathcal{L}(\mathcal{O}')$ = Dynkin diagram of $\mathcal{O}$ and $\mathcal{L}(\mathcal{O})$ = Dynkin diagram of $\mathcal{O}'$.*

Proof: The equality $\mathcal{L}(\mathcal{O})$ = Dynkin diagram of $\mathcal{O}'$ follows immediately from Theorem 3.1.

Both nilpotent orbits $\mathcal{O}$ and $\mathcal{O}'$ being Richardson orbits are *special*, see [Spaltenstein 82], [Collingwood, McGovern 93]. N. Spaltenstein defined an order reversing involutive bijection d between the sets of special nilpotent orbits for G and $G^\vee$. It follows from the definition that $d(\mathcal{O}') = \mathcal{O}$ [Spaltenstein 82]. Consequently $d(\mathcal{O}) = \mathcal{O}'$ and equality $\mathcal{L}(\mathcal{O}')$ = Dynkin diagram of $\mathcal{O}$ follows by symmetry. □

Note that not any Richardson orbit is strongly Richardson. For example the Richardson orbit for Sp_{2n} attached to the parabolic with Levi factor $GL_1 \times Sp_{2n-2}$ is not strongly Richardson for $n > 2$ (we are grateful to the referee for showing us this example).

A nilpotent orbit is called even, if its Dynkin diagram is divisible by 2 as an element of the coweight lattice, see e.g., [Collingwood, McGovern 93].

Corollary 3.3. *Suppose the orbit $\mathcal{O}$ is even and that $\mathcal{L}(\mathcal{O})$ is divisible by 2 as an element of the weight lattice. Then $\mathcal{L}(\mathcal{O})$ is the Dynkin diagram of some nilpotent orbit $\mathcal{O}'$ for $G^\vee$ and $\mathcal{L}(\mathcal{O}')$ = Dynkin diagram of $\mathcal{O}$.*

Proof: This is clear since even nilpotent orbits are strongly Richardson, see [McGovern 89]. □

A lot of examples of this corollary can be found in the tables at the end of this paper. In fact, this is the only general statement we can prove (or even just formulate) about these tables.

It is well known that nilpotent orbits in GL_n are numbered by partitions of n: to the partition $p_1 \geq p_2 \geq \ldots$ one associates an orbit of nilpotent matrices with Jordan blocks of size $p_1, p_2, \ldots$. One finds the Dynkin diagram of the orbit $\mathcal{O} = \mathcal{O}(p_1, p_2, \ldots)$ as follows [Collingwood, McGovern 93]: order n numbers $p_1 - 1, p_1 - 3, \ldots, -p_1 + 1, p_2 - 1, \ldots, -p_2 + 1, \ldots$ in decreasing order, then the label of $i-$th vertex of the Dynkin diagram will be the difference of the $i-$th and $i+1-$th numbers.

The following lemma is well known.

Lemma 3.4. For $G = GL_n$ and a parabolic subgroup $P \subset G$, the map $m_P : T^*G/P \to \overline{\mathcal{O}(P)}$ is birational.

Proof: The isotropy group of any nilpotent element in GL_n is connected. □

Let $\mathcal{O} = \mathcal{O}(p_1, p_2, \ldots)$ be a nilpotent orbit. Let $p'_1, p'_2, \ldots$ be a partition dual to $p_1, p_2, \ldots$ and let $\mathcal{O}' = \mathcal{O}(p'_1, p'_2, \ldots)$ be the orbit dual to $\mathcal{O}$.

Theorem 3.5. The weight $\mathcal{L}(\mathcal{O})$ is equal to the Dynkin diagram of the dual orbit $\mathcal{O}'$ considered as a weight for GL_n.

Proof: It is well known that any nilpotent orbit $\mathcal{O}$ in $G = GL_n$ is a Richardson orbit and it follows from Lemma 3.4 that it is strongly Richardson. It follows from the proof of Theorem 3.2 that $\mathcal{L}(\mathcal{O})$ is the Dynkin diagram of $d(\mathcal{O})$ (here d is the Spaltenstein duality). Now the result follows from the description of d for the group GL_n in [Spaltenstein 82]. □

3.3 Remarks

(i) As we mentioned in Section 1, the calculation of the map $\mathcal{L}$ is equivalent to the calculation of canonical distinguished involutions in the affine Weyl group. For type A_n, canonical distinguished involutions were recently calculated by Nanhua Xi [Xi, 00] by a completely different method. While his result coincides with ours, he does not mention a relation with Dynkin diagrams of nilpotent orbits.

(ii) For a group G of type different from A_n, it *is not* true in general that the weight attached to a nilpotent orbit and considered as a coweight for the Langlands dual group $G^\vee$ is a Dynkin diagram for some nilpotent orbit of $G^\vee$ (see the tables at the end of this paper). Calculations made by Pramod Achar (private communication) suggest some evidence for the positive answer to the following question.

Question. Is it true that any Dynkin diagram considered as a weight for the dual group corresponds under

Lusztig's bijection to a local system on some nilpotent orbit? (Here a local system is a coherent equivariant sheaf on the nilpotent orbit such that the corresponding representation of the isotropy group factors through a finite quotient.)

4. TABLES

The tables contain results of our calculations of the map $\mathcal{L}$ for groups of small rank. We have almost complete results for groups of rank ≤ 7 (there are some gaps for groups of types B_6, C_6, B_7, C_7 and E_7) and partial results in rank 8 (for groups D_8 and E_8). The tables are organized as follows: the tables for classical groups consist of 4 columns: in the first column we give the partition identifying the nilpotent orbit $\mathcal{O}$ (see e.g., [Collingwood, McGovern 93]); in the second column we give the Dynkin diagram of $\mathcal{O}$; in the third column we give the weight $\mathcal{L}(\mathcal{O})$, and the final column contains the square of the length of $\mathcal{L}(\mathcal{O})$ (we normalize the scalar product by $(\alpha, \alpha) = 2$ for a short root α). For exceptional groups, the tables consist of 3 columns: the first column contains the Dynkin diagram of the nilpotent orbit $\mathcal{O}$; the second column contains $\mathcal{L}(\mathcal{O})$, and the third column contains the square of the length of $\mathcal{L}(\mathcal{O})$. In a few cases, we were unable to calculate $\mathcal{L}(\mathcal{O})$ but we could predict the value $\mathcal{L}(\mathcal{O})$ using duality reasoning. Such cases are marked by a question mark.

SO₅

Partition	Diagram	Weight	
(5)	2 2	0 0	0
$(3, 1^2)$	2 0	1 0	2
$(2^2, 1)$	0 1	1 2	10
(1^5)	0 0	2 2	20

SO₇

Partition	Diagram	Weight	
(7)	2 2 2	0 0 0	0
$(5, 1^2)$	2 2 0	1 0 0	2
$(3^2, 1)$	0 2 0	0 0 2	6
$(3, 2^2)$	1 0 1	0 2 0	16
$(3, 1^4)$	2 0 0	2 1 0	20
$(2^2, 1^3)$	0 1 0	1 1 2	28
(1^7)	0 0 0	2 2 2	70

SO₉

Partition	Diagram	Weight	
(9)	2 2 2 2	0 0 0 0	0
$(7, 1^2)$	2 2 2 0	1 0 0 0	2
$(5, 3, 1)$	2 0 2 0	0 0 1 0	6
$(4^2, 1)$	0 2 0 1	1 0 0 2	14
$(5, 2^2)$	2 1 0 1	0 2 0 0	16
(3^3)	0 0 2 0	0 1 1 0	18
$(5, 1^4)$	2 2 0 0	2 1 0 0	20
$(3^2, 1^3)$	0 2 0 0	2 0 0 2	24
$(3, 2^2, 1^2)$	0 1 0 1	0 2 0 2	40
$(2^4, 1)$	0 0 0 1	1 1 1 2	60
$(3, 1^6)$	2 0 0 0	2 2 1 0	70
$(2^2, 1^5)$	0 1 0 0	2 1 1 2	78
(1^9)	0 0 0 0	2 2 2 2	168

SO₁₁

Partition	Diagram	Weight	
(11)	2 2 2 2 2	0 0 0 0 0	0
$(9, 1^2)$	2 2 2 2 0	1 0 0 0 0	2
$(7, 3, 1)$	2 2 0 2 0	0 0 1 0 0	6
$(5^2, 1)$	0 2 0 2 0	0 0 0 0 2	10
$(7, 2^2)$	2 2 1 0 1	0 2 0 0 0	16
$(5, 3^2)$	2 0 0 2 0	0 1 1 0 0	18
$(7, 1^4)$	2 2 2 0 0	2 1 0 0 0	20
$(5, 3, 1^3)$	2 0 2 0 0	2 0 0 1 0	24
$(4^2, 3)$	0 1 1 0 1	0 0 1 1 0	26
$(4^2, 1^3)$	0 2 0 1 0	1 1 0 0 2	32
$(3^3, 1^2)$	0 0 2 0 0	1 0 1 1 0	36
$(5, 2^2, 1^2)$	2 1 0 1 0	0 2 0 1 0	40
$(3^2, 2^2, 1)$	0 1 0 1 0	1 1 1 1 0	60
$(5, 1^6)$	2 2 0 0 0	2 2 1 0 0	70
$(3^2, 1^5)$	0 2 0 0 0	2 2 0 0 2	74
$(3, 2^4)$	1 0 0 0 1	0 2 0 2 0	80

SO_{11} (continued)

Partition	Diagram	Weight	
$(3,2^2,1^4)$	1 0 1 0 0	2 0 2 0 2	90
$(2^4,1^3)$	0 0 0 1 0	1 1 1 1 2	110
$(3,1^8)$	2 0 0 0 0	2 2 2 1 0	168
$(2^2,1^7)$	0 1 0 0 0	2 2 1 1 2	176
(1^{11})	0 0 0 0 0	2 2 2 2 2	330

SO_{13}

Partition	Diagram	Weight	
(13)	2 2 2 2 2 2	0 0 0 0 0 0	0
$(11,1^2)$	2 2 2 2 2 0	1 0 0 0 0 0	2
$(9,3,1)$	2 2 2 0 2 0	0 0 1 0 0 0	6
$(7,5,1)$	2 0 2 0 2 0	0 0 0 0 1 0	10
$(5,7,1)$	2 0 2 0 2 0	0 0 0 0 1 0	10
$(9,2^2)$	2 2 2 1 0 1	0 2 0 0 0 0	16
$(7,3^2)$	2 2 0 0 2 0	0 1 1 0 0 0	18
$(6^2,1)$	0 2 0 2 0 1	1 0 0 0 0 2	18
$(9,1^4)$	2 2 2 2 0 0	2 1 0 0 0 0	20
$(5^2,3)$	0 2 0 0 2 0	0 1 0 0 1 0	22
$(7,3,1^3)$	2 2 0 2 0 0	2 0 0 1 0 0	24
$(5^2,1^3)$	0 2 0 2 0 0	2 0 0 0 0 2	24
$(5,4^2)$	1 0 1 1 0 1	0 0 0 2 0 0	32
$(5,3^2,1^2)$	2 0 0 2 0 0	1 0 1 1 0 0	36
$(7,2^2,1)$	2 2 1 0 1 0	0 2 0 1 0 0	40
$(4^2,3,1^2)$	0 1 1 0 1 0	1 0 0 1 1 0	44
$(5,3,2^2,1)$	2 0 1 0 1 0	1 1 1 1 0 0	60
$(3^4,1)$	0 0 0 2 0 0	0 0 2 0 0 2	60
$(4,2^2,1)$	0 2 0 0 0 1	1 1 1 0 0 2	64
$(7,1^6)$	2 2 2 0 0 0	2 2 1 0 0 0	70
$(5,3,1^5)$	2 0 2 0 0 0	2 2 0 0 1 0	74
$(5,2^4)$	2 1 0 0 0 1	0 2 0 2 0 0	80
$(4^2,1^5)$	0 2 0 1 0 0	2 1 1 0 0 2	82

SO_{13} (continued)

Partition	Diagram	Weight	
$(3^3,2^2)$	0 0 1 0 1 0	0 2 0 1 1 0	82
$(3^3,1^4)$	0 0 2 0 0 0	2 1 0 1 1 0	86
$(5,2^2,1^4)$	2 1 0 1 0 0	2 0 2 0 1 0	90
$(3^2,2^2,1^3)$	0 1 0 1 0 0	1 1 1 1 1 0	110
$(3,2^4,1^2)$	1 0 0 0 1 0	0 2 0 2 1 0	138
$(5,1^8)$	2 2 0 0 0 0	2 2 2 1 0 0	168
$(3^2,1^7)$	0 2 0 0 0 0	2 2 2 0 0 2	172
$(2^6,1)$	0 0 0 0 0 1	1 1 1 1 1 2	182
$(3,1^{10})$	2 0 0 0 0 0	2 2 2 2 1 0	330
$(3,2^2,1^6)$	1 0 1 0 0 0		
$(2^4,1^5)$	0 0 0 1 0 0		
$2^2,1^9)$	0 1 0 0 0 0		
(1^{13})	0 0 0 0 0 0	2 2 2 2 2 2	572

SO_{15}

Partition	Diagram	Weight	
(15)	2 2 2 2 2 2 2	0 0 0 0 0 0 0	0
$(13,1^2)$	2 2 2 2 2 2 0	1 0 0 0 0 0 0	2
$(11,3,1)$	2 2 2 2 0 2 0	0 0 1 0 0 0 0	6
$(9,5,1)$	2 2 0 2 0 2 0	0 0 0 0 1 0 0	10
$(7^2,1)$	0 2 0 2 0 2 0	0 0 0 0 0 0 2	14
$(11,2^2)$	2 2 2 2 1 0 1	0 2 0 0 0 0 0	16
$(9,3^2)$	2 2 2 0 0 2 0	0 1 1 0 0 0 0	18
$(11,1^4)$	2 2 2 2 2 0 0	2 1 0 0 0 0 0	20
$(7,5,3)$	2 0 2 0 0 2 0	0 1 0 0 1 0 0	22
$(7,5,1^3)$	2 0 2 0 2 0 0	2 0 0 0 0 1 0	28
$(6^2,3)$	0 2 0 1 1 0 1	0 0 1 0 0 1 0	30
$(7,4^2)$	2 1 0 1 1 0 1	0 0 0 2 0 0 0	32
(5^3)	0 0 2 0 0 2 0	0 0 0 1 1 0 0	34
$(7,3^2,1^2)$	2 2 0 0 2 0 0	1 0 1 1 0 0 0	36
$(6^2,1^3)$	0 2 0 2 0 1 0	1 1 0 0 0 0 2	36
$(9,2^2,1^2)$	2 2 2 1 0 1 0	0 2 0 1 0 0 0	40

SO_{15} (continued)

Partition	Diagram	Weight	
$(5^2, 3, 1^2)$	0 2 0 0 2 0 0	1 0 1 0 0 1 0	40
$(5, 4^2, 1^2)$	1 0 1 1 0 1 0	0 1 0 1 0 1 0	56
$(5, 3^3, 1)$	2 0 0 0 2 0 0	0 0 2 0 0 1 0	60
$(7, 3, 2^2, 1)$	2 2 0 1 0 1 0	1 1 1 1 0 0 0	60
$(5^2, 2^2, 1)$	0 2 0 1 0 1 0	1 1 1 0 0 1 0	64
$(4^2, 3^2, 1)$	0 1 0 1 0 1 0	0 0 1 1 0 0 2	68
$(9, 1^6)$	2 2 2 2 0 0 0	2 2 1 0 0 0 0	70
$(7, 3, 1^5)$	2 2 0 2 0 0 0	2 2 0 0 1 0 0	74
$(5^2, 1^5)$	0 2 0 2 0 0 0	2 2 0 0 0 0 2	78
$(7, 2^4)$	2 2 1 0 0 0 1	0 2 0 2 0 0 0	80
$(5, 3^2, 2^2)$	2 0 0 1 0 1 0	0 2 0 1 1 0 0	82
$(5, 3^2, 1^4)$	2 0 0 2 0 0 0	2 1 0 1 1 0 0	86
$(4^2, 3, 2^2)$	0 1 1 0 0 0 1	0 2 0 0 1 1 0	90
$(7, 2^2, 1^4)$	2 2 1 0 1 0 0	2 0 2 0 1 0 0	90
$(4^2, 3, 1^4)$	0 1 1 0 1 0 0	2 1 0 0 1 1 0	94
(3^5)	0 0 0 0 2 0 0	0 1 1 0 1 1 0	100
$(3^4, 1^3)$	0 0 0 2 0 0 0	2 0 0 2 0 0 2	110
$(5, 3, 2^2, 1^3)$	2 0 1 0 1 0 0	1 1 1 1 1 0 0	110
$(4^2, 2^2, 1^3)$	0 2 0 0 0 1 0	1 1 1 1 0 0 2	114
$(5, 2^4, 1^2)$	2 1 0 0 0 1 0	0 2 0 2 1 0 0	138
$(7, 1^8)$	2 2 2 0 0 0 0	2 2 2 1 0 0 0	168
$(5, 3, 1^7)$	2 0 2 0 0 0 0	2 2 2 0 0 1 0	172
$(4^2, 1^7)$	0 2 0 1 0 0 0	2 2 1 1 0 0 2	180
$(3^3, 1^6)$	0 0 2 0 0 0 0	2 2 1 0 1 1 0	184
$(5, 1^{10})$	2 2 0 0 0 0 0	2 2 2 2 1 0 0	330
$(3^2, 1^9)$	0 2 0 0 0 0 0	2 2 2 2 0 0 2	334
$(3, 1^{12})$	2 0 0 0 0 0 0	2 2 2 2 2 1 0	572
$(5, 2^2, 1^6)$	2 1 0 1 0 0 0		
$(3^3, 2^2, 1^2)$	0 0 1 0 1 0 0		
$(3^2, 2^4, 1)$	0 1 0 0 0 1 0		
$(3^2, 2^2, 1^5)$	0 1 0 1 0 0 0		
$(3, 2^6)$	1 0 0 0 0 0 1		
$(3, 2^4, 1^4)$	1 0 0 0 1 0 0		

SO_{15} (continued)

Partition	Diagram	Weight	
$(3, 2^2, 1^8)$	1 0 1 0 0 0 0		
$(2^6, 1^3)$	0 0 0 0 0 1 0		
$(2^4, 1^7)$	0 0 0 1 0 0 0		
$(2^2, 1^{11})$	0 1 0 0 0 0 0		
(1^{15})	0 0 0 0 0 0 0	2 2 2 2 2 2 2	910

SO_6

Partition	Diagram	Weight	
$(5, 1)$	2 (2, 2)	0 (0, 0)	0
(3^2)	0 (2, 2)	0 (1, 1)	2
$(3, 1^3)$	2 (0, 0)	2 (0, 0)	4
$(2^2, 1^2)$	0 (1, 1)	0 (2, 2)	8
(1^6)	0 (0, 0)	2 (2, 2)	20

SO_8

Partition	Diagram	Weight	
$(7, 1)$	2 2 (2, 2)	0 0 (0, 0)	0
$(5, 3)$	2 0 (2, 2)	0 1 (0, 0)	2
$(5, 1^3)$	2 2 (0, 0)	2 0 (0, 0)	4
$(4^2)_1$	0 2 (0, 2)	0 0 (0, 2)	4
$(4^2)_2$	0 2 (2, 0)	0 0 (2, 0)	4
$(3^2, 1^2)$	0 2 (0, 0)	1 0 (1, 1)	6
$(3, 2^2, 1)$	1 0 (1, 1)	1 1 (1, 1)	14
$(3, 1^5)$	2 0 (0, 0)	2 2 (0, 0)	20
$(2^4)_1$	0 0 (0, 2)	0 2 (0, 2)	20
$(2^4)_2$	0 0 (2, 0)	0 2 (2, 0)	20
$(2^2, 1^4)$	0 1 (0, 0)	2 0 (2, 2)	24
(1^8)	0 0 (0, 0)	2 2 (2, 2)	56

SO_{10}

Partition	Diagram	Weight	
$(9,1)$	$2\ 2\ 2\ ^{2}_{2}$	$0\ 0\ 0\ ^{0}_{0}$	0
$(7,3)$	$2\ 2\ 0\ ^{2}_{2}$	$0\ 1\ 0\ ^{0}_{0}$	2
$(7,1^3)$	$2\ 2\ 2\ ^{0}_{0}$	$2\ 0\ 0\ ^{0}_{0}$	4
(5^2)	$0\ 2\ 0\ ^{2}_{2}$	$0\ 0\ 0\ ^{1}_{1}$	4
$(5,3,1^2)$	$2\ 0\ 2\ ^{0}_{0}$	$1\ 0\ 1\ ^{0}_{0}$	6
$(4^2,1^2)$	$0\ 2\ 0\ ^{1}_{1}$	$0\ 1\ 0\ ^{1}_{1}$	10
$(3^3,1)$	$0\ 0\ 2\ ^{0}_{0}$	$0\ 0\ 2\ ^{0}_{0}$	12
$(5,2^2,1)$	$2\ 1\ 0\ ^{1}_{1}$	$1\ 1\ 1\ ^{0}_{0}$	14
$(5,1^5)$	$2\ 2\ 0\ ^{0}_{0}$	$2\ 2\ 0\ ^{0}_{0}$	20
$(3^2,2^2)$	$0\ 1\ 0\ ^{1}_{1}$	$0\ 2\ 0\ ^{1}_{1}$	20
$(3^2,1^4)$	$0\ 2\ 0\ ^{0}_{0}$	$2\ 1\ 0\ ^{1}_{1}$	22
$(3,2^2,1^3)$	$1\ 0\ 1\ ^{0}_{0}$	$1\ 1\ 1\ ^{1}_{1}$	30
$(2^4,1^2)$	$0\ 0\ 0\ ^{1}_{1}$	$0\ 2\ 0\ ^{2}_{2}$	40
$(3,1^7)$	$2\ 0\ 0\ ^{0}_{0}$	$2\ 2\ 2\ ^{0}_{0}$	56
$(2^2,1^6)$	$0\ 1\ 0\ ^{0}_{0}$	$2\ 2\ 0\ ^{2}_{2}$	60
(1^{10})	$0\ 0\ 0\ ^{0}_{0}$	$2\ 2\ 2\ ^{2}_{2}$	120

SO_{12}

Partition	Diagram	Weight	
$(11,1)$	$2\ 2\ 2\ 2\ ^{2}_{2}$	$0\ 0\ 0\ 0\ ^{0}_{0}$	0
$(9,3)$	$2\ 2\ 2\ 0\ ^{2}_{2}$	$0\ 1\ 0\ 0\ ^{0}_{0}$	2
$(9,1^3)$	$2\ 2\ 2\ 2\ ^{0}_{0}$	$2\ 0\ 0\ 0\ ^{0}_{0}$	4
$(7,5)$	$2\ 0\ 2\ 0\ ^{2}_{2}$	$0\ 0\ 0\ 1\ ^{0}_{0}$	4
$(7,3,1^2)$	$2\ 2\ 0\ 2\ ^{0}_{0}$	$1\ 0\ 1\ 0\ ^{0}_{0}$	6
$(6^2)_1$	$0\ 2\ 0\ 2\ ^{2}_{0}$	$0\ 0\ 0\ 0\ ^{0}_{2}$	6
$(6^2)_2$	$0\ 2\ 0\ 2\ ^{0}_{2}$	$0\ 0\ 0\ 0\ ^{2}_{0}$	6

SO_{12} (continued)

Partition	Diagram	Weight	
$(5^2,1^2)$	$0\ 2\ 0\ 2\ ^{0}_{0}$	$1\ 0\ 0\ 0\ ^{1}_{1}$	8
$(5,3^2,1)$	$2\ 0\ 0\ 2\ ^{0}_{0}$	$0\ 0\ 2\ 0\ ^{0}_{0}$	12
$(7,2^2,1)$	$2\ 2\ 1\ 0\ ^{1}_{1}$	$1\ 1\ 1\ 0\ ^{0}_{0}$	14
$(4^2,3,1)$	$0\ 1\ 1\ 0\ ^{1}_{1}$	$0\ 0\ 0\ 2\ ^{0}_{0}$	16
$(7,1^5)$	$2\ 2\ 2\ 0\ ^{0}_{0}$	$2\ 2\ 0\ 0\ ^{0}_{0}$	20
$(5,3,2^2)$	$2\ 0\ 1\ 0\ ^{1}_{1}$	$0\ 2\ 0\ 1\ ^{0}_{0}$	20
$(4^2,2^2)_1$	$0\ 2\ 0\ 0\ ^{2}_{0}$	$0\ 2\ 0\ 0\ ^{0}_{2}$	22
$(4^2,2^2)_2$	$0\ 2\ 0\ 0\ ^{0}_{2}$	$0\ 2\ 0\ 0\ ^{2}_{0}$	22
$(5,3,1^4)$	$2\ 0\ 2\ 0\ ^{0}_{0}$	$2\ 1\ 0\ 1\ ^{0}_{0}$	22
(3^4)	$0\ 0\ 0\ 2\ ^{0}_{0}$	$0\ 1\ 1\ 0\ ^{1}_{1}$	24
$(4^2,1^4)$	$0\ 2\ 0\ 1\ ^{0}_{0}$	$2\ 0\ 1\ 0\ ^{1}_{1}$	26
$(3^3,1^3)$	$0\ 0\ 2\ 0\ ^{0}_{0}$	$2\ 0\ 0\ 2\ ^{0}_{0}$	28
$(5,2^2,1^3)$	$2\ 1\ 0\ 1\ ^{0}_{0}$	$1\ 1\ 1\ 1\ ^{0}_{0}$	30
$(3^2,2^2,1^2)$	$0\ 1\ 0\ 1\ ^{0}_{0}$	$0\ 2\ 0\ 2\ ^{0}_{0}$	40
$(3,2^4,1)$	$1\ 0\ 0\ 0\ ^{1}_{1}$	$1\ 1\ 1\ 2\ ^{0}_{0}$	54
$(5,1^7)$	$2\ 2\ 0\ 0\ ^{0}_{0}$	$2\ 2\ 2\ 0\ ^{0}_{0}$	56
$(3^2,1^6)$	$0\ 2\ 0\ 0\ ^{0}_{0}$	$2\ 2\ 1\ 0\ ^{1}_{1}$	58
$(3,2^2,1^5)$	$1\ 0\ 1\ 0\ ^{0}_{0}$	$2\ 1\ 1\ 1\ ^{1}_{1}$	66
$(2^6)_1$	$0\ 0\ 0\ 0\ ^{2}_{0}$	$0\ 2\ 0\ 2\ ^{0}_{2}$	70
$(2^6)_2$	$0\ 0\ 0\ 0\ ^{0}_{2}$	$0\ 2\ 0\ 2\ ^{2}_{0}$	70
$(2^4,1^4)$	$0\ 0\ 0\ 1\ ^{0}_{0}$	$2\ 0\ 2\ 0\ ^{2}_{2}$	76
$(3,1^9)$	$2\ 0\ 0\ 0\ ^{0}_{0}$	$2\ 2\ 2\ 2\ ^{0}_{0}$	120
$(2^2,1^8)$	$0\ 1\ 0\ 0\ ^{0}_{0}$	$2\ 2\ 2\ 0\ ^{2}_{2}$?	124
(1^{12})	$0\ 0\ 0\ 0\ ^{0}_{0}$	$2\ 2\ 2\ 2\ ^{2}_{2}$	220

SO$_{14}$

Partition	Diagram	Weight	
$(13, 1)$	2 2 2 2 2 ; 2/2	0 0 0 0 0 ; 0/0	0
$(11, 3)$	2 2 2 2 0 ; 2/2	0 1 0 0 0 ; 0/0	2
$(11, 1^3)$	2 2 2 2 2 ; 0/0	2 0 0 0 0 ; 0/0	4
$(9, 5)$	2 2 0 2 0 ; 2/2	0 0 0 1 0 ; 0/0	4
$(9, 3, 1^2)$	2 2 2 0 2 ; 0/0	1 0 1 0 0 ; 0/0	6
(7^2)	0 2 0 2 0 ; 2/2	0 0 0 0 0 ; 1/1	6
$(7, 5, 1^2)$	2 0 2 0 2 ; 0/0	1 0 0 0 1 ; 0/0	8
$(7, 3^2, 1)$	2 2 0 0 2 ; 0/0	0 0 2 0 0 ; 0/0	12
$(6^2, 1^2)$	0 2 0 2 0 ; 1/1	0 1 0 0 0 ; 1/1	12
$(9, 2^2, 1)$	2 2 2 1 0 ; 1/1	1 1 1 0 0 ; 0/0	14
$(5^2, 3, 1)$	0 2 0 0 2 ; 0/0	0 0 1 0 1 ; 0/0	14
$(9, 1^5)$	2 2 2 2 0 ; 0/0	2 2 0 0 0 ; 0/0	20
$(7, 3, 2^2)$	2 2 0 1 0 ; 1/1	0 2 0 1 0 ; 0/0	20
$(5^2, 2^2)$	0 2 0 1 0 ; 1/1	0 2 0 0 0 ; 1/1	22
$(7, 3, 1^4)$	2 2 0 2 0 ; 0/0	2 1 0 1 0 ; 0/0	22
$(5, 4^2, 1)$	1 0 1 1 0 ; 1/1	1 0 0 1 1 ; 0/0	22
$(5^2, 1^4)$	0 2 0 2 0 ; 0/0	2 1 0 0 0 ; 1/1	24
$(5, 3^3)$	2 0 0 0 2 ; 0/0	0 1 1 0 1 ; 0/0	24
$(5, 3^2, 1^3)$	2 0 0 2 0 ; 0/0	2 0 0 2 0 ; 0/0	28
$(4^2, 3^2)$	0 1 0 1 0 ; 1/1	0 1 0 1 0 ; 1/1	28
$(7, 2^2, 1^3)$	2 2 1 0 1 ; 0/0	1 1 1 1 0 ; 0/0	30
$(4^2, 3, 1^3)$	0 1 1 0 1 ; 0/0	2 0 0 0 2 ; 0/0	32
$(5, 3, 2^2, 1^2)$	2 0 1 0 1 ; 0/0	0 2 0 2 0 ; 0/0	40
$(3^4, 1^2)$	0 0 0 2 0 ; 0/0	1 0 1 1 0 ; 1/1	40
$(4^2, 2^2, 1^2)$	0 2 0 0 0 ; 1/1	0 2 0 1 0 ; 1/1	42

SO$_{14}$ **(continued)**

Partition	Diagram	Weight	
$(5, 2^4, 1)$	2 1 0 0 0 ; 1/1	1 1 1 2 0 ; 0/0	54
$(7, 1^7)$	2 2 2 0 0 ; 0/0	2 2 2 0 0 ; 0/0	56
$(3^3, 2^2, 1)$	0 0 1 0 1 ; 0/0	1 1 1 1 0 ; 1/1	56
$(5, 3, 1^6)$	2 0 2 0 0 ; 0/0	2 2 1 0 1 ; 0/0	58
$(4^2, 1^6)$	0 2 0 1 0 ; 0/0	2 2 0 1 0 ; 1/1	62
$(3^3, 1^5)$	0 0 2 0 0 ; 0/0	2 2 0 0 2 ; 0/0	64
$(5, 2^2, 1^5)$	2 1 0 1 0 ; 0/0	2 1 1 1 1 ; 0/0	66
$(3^2, 2^4)$	0 1 0 0 0 ; 1/1	0 2 0 2 0 ; 1/1 ?	70
$(2^6, 1^2)$	0 0 0 0 0 ; 1/1	0 2 0 2 0 ; 2/2 ?	112
$(5, 1^9)$	2 2 0 0 0 ; 0/0	2 2 2 2 0 ; 0/0	120
$(3^2, 1^8)$	0 2 0 0 0 ; 0/0	2 2 2 1 0 ; 1/1	122
$(2^4, 1^6)$	0 0 0 1 0 ; 0/0	2 2 0 2 0 ; 2/2 ?	140
$(3^2, 2^2, 1^4)$	0 1 0 1 0 ; 0/0		
$(3, 2^4, 1^3)$	1 0 0 0 1 ; 0/0		
$(3, 2^2, 1^7)$	1 0 1 0 0 ; 0/0		
$(3, 1^{11})$	2 0 0 0 0 ; 0/0	2 2 2 2 2 ; 0/0	220
$(2^2, 1^{10})$	0 1 0 0 0 ; 0/0	2 2 2 2 0 ; 2/2 ?	224
(1^{14})	0 0 0 0 0 ; 0/0	2 2 2 2 2 ; 2/2	364

SO$_{16}$

Partition	Diagram	Weight	
$(15, 1)$	2 2 2 2 2 2 ; 2/2	0 0 0 0 0 0 ; 0/0	0
$(13, 3)$	2 2 2 2 2 0 ; 2/2	0 1 0 0 0 0 ; 0/0	2
$(13, 1^3)$	2 2 2 2 2 2 ; 0/0	2 0 0 0 0 0 ; 0/0	4
$(11, 5)$	2 2 2 0 2 0 ; 2/2	0 0 0 1 0 0 ; 0/0	4
$(11, 3, 1^2)$	2 2 2 2 0 2 ; 0/0	1 0 1 0 0 0 ; 0/0	6

SO_{16} (continued)

Partition	Diagram	Weight	
$(9,7)$	2 0 2 0 2 0 | 2 2	0 0 0 0 0 1 | 0 0	6
$(9,5,1^2)$	2 2 0 2 0 2 | 0 0	1 0 0 0 1 0 | 0 0	8
$(8^2)_1$	0 2 0 2 0 2 | 0 2	0 0 0 0 0 0 | 0 2	8
$(8^2)_2$	0 2 0 2 0 2 | 2 0	0 0 0 0 0 0 | 2 0	8
$(7^2,1^2)$	0 2 0 2 0 2 | 0 0	1 0 0 0 0 0 | 1 1	10
$(9,3^2,1)$	2 2 2 0 0 2 | 0 0	0 0 2 0 0 0 | 0 0	12
$(11,2^2,1)$	2 2 2 2 1 0 | 1 1	1 1 1 0 0 0 | 0 0	14
$(7,5,3,1)$	2 0 2 0 0 2 | 0 0	0 0 1 0 1 0 | 0 0	14
$(6^2,3,1)$	0 2 0 1 1 0 | 1 1	0 0 0 1 0 1 | 0 0	18
$(11,1^5)$	2 2 2 2 2 0 | 0 0	2 2 0 0 0 0 | 0 0	20
$(9,3,2^2)$	2 2 2 0 1 0 | 1 1	0 2 0 1 0 0 | 0 0	20
$(5^3,1)$	0 0 2 0 0 2 | 0 0	0 0 0 0 2 0 | 0 0	20
$(9,3,1^4)$	2 2 2 0 2 0 | 0 0	2 1 0 1 0 0 | 0 0	22
$(7,5,2^2)$	2 0 2 0 1 0 | 1 1	0 2 0 0 0 1 | 0 0	22
$(7,4^2,1)$	2 1 0 1 1 0 | 1 1	1 0 0 1 1 0 | 0 0	22
$(7,5,1^4)$	2 0 2 0 2 0 | 0 0	2 1 0 0 0 1 | 0 0	24
$(7,3^3)$	2 2 0 0 0 2 | 0 0	0 1 1 0 1 0 | 0 0	24
$(6^2,2^2)_1$	0 2 0 2 0 0 | 0 2	0 2 0 0 0 0 | 0 2	24
$(6^2,2^2)_2$	0 2 0 2 0 0 | 2 0	0 2 0 0 0 0 | 2 0	24
$(5^2,3^2)$	0 2 0 0 0 2 | 0 0	0 1 1 0 0 0 | 1 1	26
$(7,3^2,1^3)$	2 2 0 0 2 0 | 0 0	2 0 0 2 0 0 | 0 0	28
$(6^2,1^4)$	0 2 0 2 0 1 | 0 0	2 0 1 0 0 0 | 1 1	28
$(5^2,3,1^3)$	0 2 0 0 2 0 | 0 0	2 0 0 1 0 1 | 0 0	30
$(9,2^2,1^3)$	2 2 2 1 0 1 | 0 0	1 1 1 1 0 0 | 0 0	30
$(5,4^2,3)$	1 0 1 0 1 0 | 1 1	0 0 1 1 0 0 | 1 1	34
$(5,4^2,1^3)$	1 0 1 1 0 1 | 0 0	1 1 0 0 1 1 | 0 0	38
$(7,3,2^2,1^2)$	2 2 0 1 0 1 | 0 0	0 2 0 2 0 0 | 0 0	40
$(5,3^3,1^2)$	2 0 0 0 2 0 | 0 0	1 0 1 1 0 1 | 0 0	40
$(4^4)_1$	0 0 0 2 0 0 | 0 2	0 0 0 2 0 0 | 0 2	40
$(4^4)_2$	0 0 0 2 0 0 | 2 0	0 0 0 2 0 0 | 2 0	40
$(5^2,2^2,1^2)$	0 2 0 1 0 1 | 0 0	0 2 0 1 0 1 | 0 0	42
$(7,2^4,1)$	2 2 1 0 0 0 | 1 1	1 1 1 2 0 0 | 0 0	54
$(9,1^7)$	2 2 2 2 0 0 | 0 0	2 2 2 0 0 0 | 0 0	56
$(5,3^2,2^2,1)$	2 0 0 1 0 1 | 0 0	1 1 1 1 0 1 | 0 0	56
$(7,3,1^6)$	2 2 0 2 0 0 | 0 0	2 2 1 0 1 0 | 0 0	58
$(4^2,3,2^2,1)$	0 1 1 0 0 0 | 1 1	1 1 1 0 1 0 | 1 1	60
$(5^2,1^6)$	0 2 0 2 0 0 | 0 0	2 2 1 0 0 0 | 1 1	60
$(3^5,1)$	0 0 0 0 2 0 | 0 0	0 0 2 0 0 2 | 0 0	60
$(5,3^2,1^5)$	2 0 0 2 0 0 | 0 0	2 2 0 0 2 0 | 0 0	64
$(7,2^2,1^5)$	2 2 1 0 1 0 | 0 0	2 1 1 1 1 0 | 0 0	66
$(5,3,2^4)$	2 0 1 0 0 0 | 1 1	0 2 0 2 0 1 | 0 0	70
$(4^2,2^4)_1$	0 2 0 0 0 0 | 0 2	0 2 0 2 0 0 | 0 2	72
$(4^2,2^4)_2$	0 2 0 0 0 0 | 2 0	0 2 0 2 0 0 | 2 0	72
$(3^4,1^4)$	0 0 0 2 0 0 | 0 0	2 1 0 1 1 0 | 1 1	76
$(4^2,2^2,1^4)$	0 2 0 0 0 1 | 0 0	2 0 2 0 0 2 | 0 0	80
$(7,1^9)$	2 2 2 0 0 0 | 0 0	2 2 2 2 0 0 | 0 0	120
$(5,3,1^8)$	2 0 2 0 0 0 | 0 0	2 2 2 1 0 1 | 0 0	122
$(4^2,1^8)$	0 2 0 1 0 0 | 0 0	2 2 2 0 1 0 | 1 1	126
$(3^3,1^7)$	0 0 2 0 0 0 | 0 0	2 2 2 0 0 2 | 0 0	128
$(2^8)_1$	0 0 0 0 0 0 | 0 2	0 2 0 2 0 2 | 0 2	168

$\mathbf{so}_{16}$ (continued)

Partition	Diagram	Weight	
$(2^8)_2$	0 0 0 0 0 0 (2, 0)	0 2 0 2 0 2 (2?, 0)	168
$(5, 1^{11})$	2 2 0 0 0 0 (0, 0)	2 2 2 2 2 0 (0?, 0)	220
$(3^2, 1^{10})$	0 2 0 0 0 0 (0, 0)	2 2 2 2 1 0 (1, 1)	222
$(3, 1^{13})$	2 0 0 0 0 0 (0, 0)	2 2 2 2 2 2 (0, 0)	364
$(5, 3, 2^2, 1^4)$	2 0 1 0 1 0 (0, 0)		
$(5, 2^4, 1^3)$	2 1 0 0 0 1 (0, 0)		
$(5, 2^2, 1^7)$	2 1 0 1 0 0 (0, 0)		
$(4^2, 3^2, 1^2)$	0 1 0 1 0 1 (0, 0)		
$(4^2, 3, 1^5)$	0 1 1 0 1 0 (0, 0)		
$(3^4, 2^2)$	0 0 0 1 0 1 (0, 0)		
$(3^3, 2^2, 1^3)$	0 0 1 0 1 0 (0, 0)		
$(3^2, 2^4, 1^2)$	0 1 0 0 0 1 (0, 0)		
$(3^2, 2^2, 1^6)$	0 1 0 1 0 0 (0, 0)		
$(3, 2^6, 1)$	1 0 0 0 0 0 (1, 1)		
$(3, 2^4, 1^5)$	1 0 0 0 1 0 (0, 0)		
$(3, 2^2, 1^9)$	1 0 1 0 0 0 (0, 0)		
$(2^6, 1^4)$	0 0 0 0 0 1 (0, 0)		
$(2^4, 1^8)$	0 0 0 1 0 0 (0, 0)		
$(2^2, 1^{12})$	0 1 0 0 0 0 (0, 0)		
(1^{16})	0 0 0 0 0 0 (0, 0)	2 2 2 2 2 2 (2, 2)	560

$\mathbf{sp}_4$

Partition	Diagram	Weight	
(4)	2 2	0 0	0
(2^2)	0 2	0 1	2
$(2, 1^2)$	1 0	2 1	10
(1^4)	0 0	2 2	20

$\mathbf{sp}_6$

Partition	Diagram	Weight	
(6)	2 2 2	0 0 0	0
$(4, 2)$	2 0 2	0 1 0	2
(3^2)	0 2 0	1 0 1	6
(2^3)	0 0 2	0 2 0	8
$(4, 1^2)$	2 1 0	2 1 0	10
$(2^2, 1^2)$	0 1 0	2 2 0	20
$(2, 1^4)$	1 0 0	2 3 0	34
(1^6)	0 0 0	2 2 2	56

$\mathbf{sp}_8$

Partition	Diagram	Weight	
(8)	2 2 2 2	0 0 0 0	0
$(6, 2)$	2 2 0 2	0 1 0 0	2
(4^2)	0 2 0 2	0 0 0 1	4
$(4, 2^2)$	2 0 0 2	0 2 0 0	8
$(6, 1^2)$	2 2 1 0	2 1 0 0	10
$(3^2, 2)$	0 1 1 0	0 0 2 0	12
$(4, 2, 1^2)$	2 0 1 0	2 2 0 0	20
(2^4)	0 0 0 2	0 2 0 1	20
$(3^2, 1^2)$	0 2 0 0	2 1 0 1	22
$(4, 1^4)$	2 1 0 0	2 3 0 0	34
$(2^3, 1^2)$	0 0 1 0	2 2 0 1	36
$(2^2, 1^4)$	0 1 0 0	2 3 0 1	54
$(2, 1^6)$	1 0 0 0	2 2 2 1	84
(1^8)	0 0 0 0	2 2 2 2	120

$\mathbf{sp}_{10}$

Partition	Diagram	Weight	
(10)	2 2 2 2 2	0 0 0 0 0	0
$(8, 2)$	2 2 2 0 2	0 1 0 0 0	2
$(6, 4)$	2 0 2 0 2	0 0 0 1 0	4
$(6, 2^2)$	2 2 0 0 2	0 2 0 0 0	8
(5^2)	0 2 0 2 0	1 0 0 0 1	8

$\mathbf{sp}_{10}$ (continued)

Partition	Diagram	Weight	
$(4^2, 2)$	0 2 0 0 2	0 1 0 1 0	10
$(8, 1^2)$	2 2 2 1 0	2 1 0 0 0	10
$(4, 3^2)$	1 0 1 1 0	1 0 1 1 0	18
$(6, 2, 1^2)$	2 2 0 1 0	2 2 0 0 0	20
$(4, 2^3)$	2 0 0 0 2	0 2 0 1 0	20
$(4^2, 1^2)$	0 2 0 1 0	2 1 0 1 0	22
$(3^2, 2^2)$	0 1 0 1 0	0 1 1 0 1	24
$(6, 1^4)$	2 2 1 0 0	2 3 0 0 0	34
$(4, 2^2, 1^2)$	2 0 0 1 0	2 2 0 1 0	36
$(3^2, 2, 1^2)$	0 1 1 0 0	2 1 1 0 1	40
(2^5)	0 0 0 0 2	0 2 0 2 0	40
$(4, 2, 1^4)$	2 0 1 0 0	2 3 0 1 0	54
$(3^2, 1^4)$	0 2 0 0 0	2 2 1 0 1	58
$(2^4, 1^2)$	0 0 0 1 0	2 2 0 2 0	60
$(2^3, 1^4)$	0 0 1 0 0	2 3 0 2 0	82
$(4, 1^6)$	2 1 0 0 0	2 2 2 1 0	84
$(2^2, 1^6)$	0 1 0 0 0	2 2 2 2 0	120
$(2, 1^8)$	1 0 0 0 0	2 2 2 3 0	164
(1^{10})	0 0 0 0 0	2 2 2 2 2	220

$\mathbf{sp}_{12}$

Partition	Diagram	Weight	
(12)	2 2 2 2 2 2	0 0 0 0 0 0	0
$(10, 2)$	2 2 2 2 0 2	0 1 0 0 0 0	2
$(8, 4)$	2 2 0 2 0 2	0 0 0 1 0 0	4
(6^2)	0 2 0 2 0 2	0 0 0 0 0 1	6
$(8, 2^2)$	2 2 2 0 0 2	0 2 0 0 0 0	8
$(10, 1^2)$	2 2 2 2 1 0	2 1 0 0 0 0	10
$(6, 4, 2)$	2 0 2 0 0 2	0 1 0 1 0 0	10
$(5^2, 2)$	0 2 0 1 1 0	0 0 1 0 1 0	14

$\mathbf{sp}_{12}$ (continued)

Partition	Diagram	Weight	
(4^3)	0 0 2 0 0 2	0 0 0 2 0 0	16
$(6, 3^2)$	2 1 0 1 1 0	1 0 1 1 0 0	18
$(6, 2^3)$	2 2 0 0 0 2	0 2 0 1 0 0	20
$(8, 2, 1^2)$	2 2 2 1 0 1	2 2 0 0 0 0	20
$(6, 4, 1^2)$	2 0 2 0 1 0	2 1 0 1 0 0	22
$(4^2, 2^2)$	0 2 0 0 0 2	0 2 0 0 0 1	22
$(5^2, 1^2)$	0 2 0 2 0 0	2 1 0 0 0 1	24
$(8, 1^4)$	2 2 2 1 0 0	2 3 0 0 0 0	34
$(6, 2^2, 1^2)$	2 2 0 0 1 0	2 2 0 1 0 0	36
$(4, 2^2)$	2 0 0 0 0 2	0 2 0 2 0 0	40
(3^4)	0 0 0 2 0 0	1 0 1 1 0 1	40
$(4, 3^2, 1^2)$	1 0 1 1 0 0	2 1 0 2 0 0	42
$(3^2, 2^3)$	0 1 0 0 1 0	0 2 0 0 2 0	44
$(6, 2, 1^4)$	2 2 0 1 0 0	2 3 0 1 0 0	54
$(4^2, 1^4)$	0 2 0 1 0 0	2 3 0 0 0 1	56
$(4, 3^2, 2)$	1 0 1 0 1 0	0 0 2 0 0 2	60
$(4, 2^3, 1^2)$	2 0 0 0 1 0	2 2 0 2 0 0	60
$(3^2, 2^2, 1^2)$	0 1 0 1 0 0	2 2 0 0 2 0	64
(2^6)	0 0 0 0 0 2	0 2 0 2 0 1	70
$(4, 2^2, 1^4)$	2 0 0 1 0 0	2 3 0 2 0 0	82
$(6, 1^6)$	2 2 1 0 0 0	2 2 2 1 0 0	84
$(3^2, 2, 1^4)$	0 1 1 0 0 0	2 3 0 0 2 0	86
$(2^5, 1^2)$	0 0 0 0 1 0	2 2 0 2 0 1	94
$(4, 2, 1^6)$	2 0 1 0 0 0	2 2 2 2 0 0	120
$(3^2, 1^6)$	0 2 0 0 0 0	2 2 2 1 0 1	122
$(4, 1^8)$	2 1 0 0 0 0	2 2 2 3 0 0	164
$(2^4, 1^4)$	0 0 0 1 0 0		
$(2^3, 1^6)$	0 0 1 0 0 0		
$(2^2, 1^8)$	0 1 0 0 0 0		
$(2, 1^{10})$	1 0 0 0 0 0		
(1^{12})	0 0 0 0 0 0	2 2 2 2 2 2	364

$\mathbf{sp}_{14}$

Partition	Diagram	Weight	
(14)	2 2 2 2 2 2 2	0 0 0 0 0 0 0	0
$(12,2)$	2 2 2 2 2 0 2	0 1 0 0 0 0 0	2
$(10,4)$	2 2 2 0 2 0 2	0 0 0 1 0 0 0	4
$(8,6)$	2 0 2 0 2 0 2	0 0 0 0 0 1 0	6
$(10,2^2)$	2 2 2 2 0 0 2	0 2 0 0 0 0 0	8
$(12,1^2)$	2 2 2 2 2 1 0	2 1 0 0 0 0 0	10
$(8,4,2)$	2 2 0 2 0 0 2	0 1 0 1 0 0 0	10
(7^2)	0 2 0 2 0 2 0	1 0 0 0 0 0 1	10
$(6^2,2)$	0 2 0 2 0 0 2	0 1 0 0 0 1 0	12
$(6,4^2)$	2 0 0 2 0 0 2	0 0 0 2 0 0 0	16
$(8,3^2)$	2 2 1 0 1 1 0	1 0 1 1 0 0 0	18
$(10,2,1^2)$	2 2 2 2 0 1 0	2 2 0 0 0 0 0	20
$(8,2^3)$	2 2 2 0 0 0 2	0 2 0 1 0 0 0	20
$(5^2,4)$	0 1 1 0 1 1 0	0 0 0 0 2 0 0	20
$(8,4,1^2)$	2 2 0 2 0 1 0	2 1 0 1 0 0 0	22
$(6,4,2^2)$	2 0 2 0 0 0 2	0 2 0 0 0 1 0	22
$(6^2,1^2)$	0 2 0 2 0 1 0	2 1 0 0 0 1 0	24
$(5^2,2^2)$	0 2 0 1 0 1 0	0 1 1 0 0 0 1	26
$(4^3,2)$	0 0 2 0 0 0 2	0 1 0 1 0 1 0	28
$(6,3^2,2)$	2 1 0 1 0 1 0	0 0 2 0 0 1 0	30
$(10,1^4)$	2 2 2 2 1 0 0	2 3 0 0 0 0 0	34
$(8,2^2,1^2)$	2 2 2 0 0 1 0	2 2 0 1 0 0 0	36
$(6,4,2,1^2)$	2 0 2 0 0 1 0	2 2 0 0 0 1 0	38
$(6,2^4)$	2 2 0 0 0 0 2	0 2 0 2 0 0 0	40
$(4^2,3^2)$	0 1 0 1 0 1 0	1 0 1 1 0 1 0	40
$(4^2,2^3)$	0 2 0 0 0 0 2	0 2 0 1 0 1 0	42
$(6,3^2,1^2)$	2 1 0 1 1 0 0	2 1 0 2 0 0 0	42
$(5^2,2,1^2)$	0 2 0 1 1 0 0	2 1 1 0 0 0 1	42
$(4^3,1^2)$	0 0 2 0 0 1 0	2 1 0 1 0 1 0	44
$(4,3^2,2^2)$	1 0 1 0 0 1 0	0 1 1 0 1 1 0	50
$(8,2,1^4)$	2 2 2 0 1 0 0	2 3 0 1 0 0 0	54
$(6,4,1^4)$	2 0 2 0 1 0 0	2 3 0 0 0 1 0	56

$\mathbf{sp}_{14}$ (continued)

Partition	Diagram	Weight	
$(6,2^3,1^2)$	2 2 0 0 0 1 0	2 2 0 2 0 0 0	60
$(5^2,1^4)$	0 2 0 2 0 0 0	2 2 1 0 0 0 1	60
$(3^4,2)$	0 0 0 1 1 0 0	0 0 2 0 0 2 0	60
$(4^2,2^2,1^2)$	0 2 0 0 0 1 0	2 2 0 1 0 1 0	62
$(4,3^2,2,1^2)$	1 0 1 0 1 0 0	2 1 1 0 1 1 0	70
$(4,2^5)$	2 0 0 0 0 0 2	0 2 0 2 0 1 0	70
$(3^4,1^2)$	0 0 0 2 0 0 0	2 1 0 1 1 0 1	76
$(6,2^2,1^4)$	2 2 0 0 1 0 0	2 3 0 2 0 0 0	82
$(4^2,2,1^4)$	0 2 0 0 1 0 0	2 3 0 1 0 1 0	84
$(8,1^6)$	2 2 2 1 0 0 0	2 2 2 1 0 0 0	84
$(4,3^2,1^4)$	1 0 1 1 0 0 0	2 2 1 0 1 1 0	92
$(4,2^4,1^2)$	2 0 0 0 0 1 0	2 2 0 2 0 1 0	94
$(6,2,1^6)$	2 2 0 1 0 0 0	2 2 2 2 0 0 0	120
(2^7)	0 0 0 0 0 0 2	0 2 0 2 0 2 0	112
$(4^2,1^6)$	0 2 0 1 0 0 0	2 2 2 1 0 1 0	122
$(6,1^8)$	2 2 1 0 0 0 0	2 2 2 3 0 0 0	164
$(3^2,1^8)$	0 2 0 0 0 0 0	2 2 2 2 1 0 1	222
$(4,2^3,1^4)$	2 0 0 0 1 0 0		
$(4,2^2,1^6)$	2 0 0 1 0 0 0		
$(4,2,1^8)$	2 0 1 0 0 0 0		
$(4,1^10)$	2 1 0 0 0 0 0		
$(3^2,2^4)$	0 1 0 0 0 1 0		
$(3^2,2^3,1^2)$	0 1 0 0 1 0 0		
$(3^2,2^2,1^4)$	0 1 0 1 0 0 0		
$(3^2,2,1^6)$	0 1 1 0 0 0 0		
$(2^6,1^2)$	0 0 0 0 0 1 0		
$(2^5,1^4)$	0 0 0 0 1 0 0		
$(2^4,1^6)$	0 0 0 1 0 0 0		
$(2^3,1^8)$	0 0 1 0 0 0 0		
$(2^2,1^{10})$	0 1 0 0 0 0 0		
$(2,1^{12})$	1 0 0 0 0 0 0		
(1^{14})	0 0 0 0 0 0 0	2 2 2 2 2 2 2	560

The Exceptional Groups

$\mathbf{G}_2$

Diagram	Weight	
2 2	0 0	0
0 2	1 0	2
1 0	1 1	14
0 1	3 0	18
0 0	2 2	56

$\mathbf{F}_4$

Diagram	Weight	
2 2 2 2	0 0 0 0	0
2 2 0 2	0 0 0 1	2
0 2 0 2	0 0 1 0	6
2 2 0 0	0 0 0 2	8
0 2 0 0	0 1 0 0	12
1 0 1 2	2 0 0 0	16
1 0 1 0	1 1 0 0	28
0 1 0 1	0 1 1 0	34
2 0 0 1	1 0 0 3	34
0 0 1 0	0 0 2 1	38
2 0 0 0	0 0 2 2	56
0 0 0 2	2 1 0 1	70
0 1 0 0	2 0 2 0	72
0 0 0 1	2 1 0 3	118
1 0 0 0	1 1 2 2	156
0 0 0 0	2 2 2 2	312

$\mathbf{E}_6$

Diagram	Weight	
2 2 2 2 2 [2]	0 0 0 0 0 [0]	0
2 2 0 2 2 [2]	0 0 0 0 0 [1]	2
2 0 2 0 2 [2]	1 0 0 0 1 [0]	4
2 0 2 0 2 [0]	0 0 1 0 0 [0]	6
1 1 0 1 1 [2]	1 0 0 0 1 [1]	10
1 1 0 1 1 [1]	0 1 0 1 0 [0]	12
2 1 0 1 2 [1]	0 0 1 0 0 [1]	14
0 0 2 0 0 [2]	2 0 0 0 2 [0]	16
0 0 2 0 0 [0]	1 0 1 0 1 [0]	18
2 0 0 0 2 [2]	1 0 0 0 1 [2]	20
0 1 0 1 0 [1]	1 0 1 0 1 [1]	30
1 0 1 0 1 [0]	0 1 1 1 0 [0]	34
1 0 0 0 1 [2]	2 0 0 0 2 [2]	40

$\mathbf{E}_6$ (continued)

Diagram	Weight	
0 1 0 1 0 [0]	1 1 0 1 1 [1]	42
2 0 0 0 2 [0]	0 0 2 0 0 [2]	56
1 0 0 0 1 [1]	1 1 0 1 1 [2]	60
0 0 0 0 0 [2]	2 1 0 1 2 [1]	70
0 0 1 0 0 [0]	1 1 1 1 1 [1]	78
1 0 0 0 1 [0]	2 0 2 0 2 [2]	120
0 0 0 0 0 [1]	2 2 0 2 2 [2]	168
0 0 0 0 0 [0]	2 2 2 2 2 [2]	312

$\mathbf{E}_7$

Diagram	Weight	
2 2 2 2 2 2 [2]	0 0 0 0 0 0 [0]	0
2 2 0 2 2 2 [2]	1 0 0 0 0 0 [0]	2
2 2 0 2 0 2 [2]	0 0 0 0 1 0 [0]	4
2 0 2 0 2 2 [0]	0 1 0 0 0 0 [0]	6

$\mathbf{E}_7$ (continued)

Diagram	Weight	
(0) 2 2 2 0 2 0	(0) 0 0 0 0 0 2	6
(0) 2 0 2 0 2 0	(1) 0 0 0 0 0 1	8
(0) 2 0 2 0 0 2	(0) 0 0 1 0 0 0	12
(1) 2 1 0 1 2 2	(0) 1 1 0 0 0 0	14
(0) 0 0 2 0 2 0	(2) 0 0 0 0 0 0	14
(1) 2 1 0 1 1 0	(0) 0 0 0 0 2 0	16
(0) 0 0 2 0 0 2	(0) 0 1 0 0 1 0	18
(1) 2 1 0 1 0 2	(0) 2 0 0 0 1 0	20
(0) 2 2 0 0 2 0	(0) 2 0 0 0 0 2	22
(0) 0 2 0 0 2 0	(0) 1 0 0 1 0 1	24
(0) 2 0 0 2 0 0	(0) 0 0 1 0 1 0	28
(1) 0 1 0 1 0 2	(0) 1 1 0 0 1 0	30
(0) 0 0 2 0 0 0	(0) 0 0 0 2 0 0	30
(0) 1 0 1 0 2 0	(1) 1 0 0 1 0 0	32

$\mathbf{E}_7$ (continued)

Diagram	Weight	
(0) 1 0 1 0 1 2	(0) 0 1 1 0 0 0	34
(0) 2 0 1 0 1 0	(0) 2 0 0 0 2 0	40
(0) 1 0 1 0 1 0	(0) 1 0 1 0 1 0	42
(0) 0 0 0 2 0 0	(0) 0 0 2 0 0 0	48
(1) 2 1 0 0 0 1	(0) 1 1 0 0 2 0	54
(0) 2 0 0 0 2 2	(0) 2 2 0 0 0 0	56
(0) 2 0 0 0 2 0	(1) 2 1 0 0 0 1	58
(0) 0 0 1 0 1 0	(0) 2 0 0 2 0 0	62
(1) 0 1 0 0 0 1	(0) 1 0 1 0 2 0	70
(0) 2 2 0 0 0 0	(0) 2 0 0 0 2 2	70
(0) 0 2 0 0 0 0	(0) 1 0 1 0 1 2	72
(0) 1 0 0 1 0 1	(0) 1 1 1 0 1 0	78
(0) 1 0 1 0 0 0	(1) 1 0 1 0 1 1	84
(0) 0 1 0 0 1 0	(0) 0 1 1 1 0 1	88

$\mathbf{E}_7$ (continued)

Diagram	Weight	
2 / 0 0 0 0 0 0	0 / 0 0 2 0 2 0	112
0 / 2 0 0 0 0 2	0 / 2 2 0 0 2 0	120
0 / 0 0 0 0 2 0	1 / 2 1 0 1 1 0	122
0 / 2 0 0 0 1 0	1 / 2 1 0 1 0 2	124
0 / 0 0 1 0 0 0	0 / 2 0 2 0 0 2	126
0 / 1 0 0 0 1 0	1 / 2 1 0 1 2 0	166
1 / 0 0 0 0 0 1	0 / 2 0 2 0 2 0 ?	168
0 / 2 0 0 0 0 0	1 / 2 1 0 1 2 2	220
0 / 0 1 0 0 0 0	0 / 2 0 2 0 2 2 ?	222
0 / 0 0 0 0 0 2	0 / 2 2 2 0 2 0	312
0 / 0 0 0 0 1 0	2 / 2 2 0 2 0 2 ?	318
0 / 1 0 0 0 0 0	2 / 2 2 0 2 2 2 ?	462
0 / 0 0 0 0 0 0	2 / 2 2 2 2 2 2	798

$\mathbf{E}_8$

Diagram	Weight	
2 / 2 2 2 2 2 2 2	0 / 0 0 0 0 0 0 0	0
2 / 2 2 0 2 2 2 2	0 / 0 0 0 0 0 0 1	2
2 / 2 2 0 2 0 2 2	0 / 1 0 0 0 0 0 0	4
0 / 2 0 2 0 2 2 2	0 / 0 0 0 0 0 1 0	6
0 / 2 0 2 0 2 0 2	1 / 0 0 0 0 0 0 0	8
0 / 2 0 2 0 0 2 2	0 / 0 0 0 0 1 0 0	12
1 / 2 1 0 1 2 2 2	0 / 0 0 0 0 0 1 1	14
0 / 2 0 2 0 0 2 0	0 / 0 1 0 0 0 0 0	14
1 / 2 1 0 1 0 2 2	0 / 1 0 0 0 0 0 2	20
1 / 2 1 0 1 1 0 1	1 / 1 0 0 0 0 0 0	22
0 / 2 0 0 2 0 0 2	0 / 1 0 0 0 1 0 0	28
1 / 0 1 0 1 0 2 2	0 / 1 0 0 0 0 1 1	30
0 / 2 0 1 0 1 0 2	0 / 2 0 0 0 0 0 2	40
0 / 1 0 1 0 1 0 2	0 / 1 0 0 0 1 0 1	42

$\mathbf{E}_8$ (continued)

(Each diagram label is given as: top node / bottom row of seven nodes.)

Diagram	Weight	
0 / 0 0 0 2 0 0 2	0 / 0 0 0 0 2 0 0	48
0 / 0 0 2 0 0 2 2	0 / 0 1 0 0 1 0 0	50
1 / 2 1 0 0 0 1 2	0 / 2 0 0 0 0 1 1	54
0 / 1 0 1 0 1 1 0		
0 / 1 0 1 0 1 0 1		
0 / 2 0 0 0 2 2 2	0 / 0 0 0 0 0 2 2	56
0 / 2 0 0 0 2 0 2	1 / 0 0 0 0 0 1 2	58
0 / 0 0 0 2 0 0 0	0 / 0 0 1 0 0 1 0	60
0 / 2 0 0 0 2 0 0	2 / 0 0 0 0 0 0 2	64
0 / 1 0 0 1 0 1 2	0 / 1 0 0 0 1 1 1	78
0 / 0 0 1 0 1 0 0		
0 / 1 0 0 1 0 1 0		
1 / 0 1 0 0 0 1 0		
0 / 0 1 0 0 1 0 1		

$\mathbf{E}_8$ (continued)

Diagram	Weight	
0 / 1 0 1 0 0 0 1		
0 / 0 0 1 0 0 1 0	0 / 0 0 0 2 0 0 0 ?	80
0 / 0 0 1 0 1 0 2	1 / 1 0 0 1 0 0 0	82
0 / 0 0 2 0 0 2 0	1 / 0 1 0 0 0 1 1	98
0 / 1 0 1 0 1 0 0		
1 / 0 1 0 0 0 1 2	0 / 1 0 1 0 0 1 1	116
0 / 0 0 2 0 0 0 2	0 / 0 0 2 0 0 1 0	174
2 / 0 0 0 0 0 0 2	0 / 2 0 0 0 2 0 0	112
0 / 2 0 0 0 0 2 2	0 / 2 0 0 0 0 2 2	120
0 / 2 0 0 0 0 2 0	0 / 1 0 0 1 0 1 2	122
0 / 0 1 0 0 1 0 0		
0 / 0 0 1 0 0 0 2	0 / 0 0 1 0 1 0 2	126
0 / 2 0 0 0 1 0 1		
0 / 0 0 0 0 2 0 0	0 / 0 0 0 2 0 0 2	128

$\mathbf{E}_8$ (continued)

Diagram	Weight	
$0\ 0\ \overset{0}{1}\ 0\ 0\ 0\ 1$		
$1\ 0\ \overset{0}{0}\ 1\ 0\ 0\ 0$		
$1\ 0\ \overset{0}{0}\ 0\ 1\ 0\ 1$		
$0\ 0\ \overset{2}{0}\ 0\ 0\ 0\ 0$	$1\ 0\ \overset{0}{1}\ 0\ 1\ 1\ 0$	168
$0\ 0\ \overset{1}{0}\ 0\ 0\ 1\ 2$	$2\ 0\ \overset{0}{0}\ 0\ 2\ 0\ 2$?	168
$0\ 0\ \overset{0}{1}\ 0\ 0\ 0\ 0$		
$2\ 0\ \overset{0}{0}\ 0\ 0\ 0\ 2$	$2\ 1\ \overset{1}{0}\ 0\ 0\ 1\ 2$	220
$1\ 0\ \overset{0}{1}\ 0\ 1\ 2\ 2$	$1\ 0\ \overset{2}{1}\ 0\ 0\ 0\ 2$	230
$1\ 0\ \overset{0}{0}\ 0\ 1\ 0\ 0$		
$0\ 0\ \overset{1}{0}\ 0\ 0\ 1\ 0$		
$0\ 1\ \overset{0}{0}\ 0\ 0\ 0\ 1$		
$0\ 0\ \overset{0}{0}\ 1\ 0\ 0\ 0$		
$0\ 0\ \overset{0}{0}\ 0\ 0\ 2\ 2$	$2\ 0\ \overset{0}{0}\ 0\ 2\ 2\ 2$	312
$0\ 0\ \overset{0}{0}\ 0\ 0\ 2\ 0$	$1\ 0\ \overset{0}{1}\ 0\ 1\ 2\ 2$	314

$\mathbf{E}_8$ (continued)

Diagram	Weight	
$1\ 0\ \overset{0}{0}\ 0\ 1\ 0\ 2$	$2\ 1\ \overset{1}{0}\ 0\ 1\ 1\ 2$	330
$0\ 0\ \overset{0}{0}\ 0\ 1\ 0\ 1$		
$1\ 0\ \overset{0}{0}\ 0\ 0\ 1\ 0$		
$2\ 0\ \overset{0}{0}\ 0\ 0\ 0\ 0$	$1\ 1\ \overset{1}{0}\ 1\ 1\ 1\ 0$	336
$0\ 1\ \overset{0}{0}\ 0\ 0\ 0\ 0$	$2\ 0\ \overset{0}{2}\ 0\ 0\ 2\ 0$?	368
$1\ 0\ \overset{0}{0}\ 0\ 0\ 0\ 2$	$2\ 1\ \overset{1}{0}\ 1\ 0\ 2\ 2$?	462
$0\ 0\ \overset{0}{0}\ 0\ 1\ 0\ 0$	$2\ 0\ \overset{0}{2}\ 0\ 0\ 2\ 2$?	464
$1\ 0\ \overset{0}{0}\ 0\ 0\ 0\ 1$		
$0\ 0\ \overset{1}{0}\ 0\ 0\ 0\ 0$	$2\ 0\ \overset{0}{2}\ 0\ 2\ 0\ 2$?	560
$0\ 0\ \overset{0}{0}\ 0\ 0\ 0\ 2$	$2\ 1\ \overset{1}{0}\ 1\ 2\ 2\ 2$	798
$0\ 0\ \overset{0}{0}\ 0\ 0\ 1\ 0$	$2\ 0\ \overset{0}{2}\ 0\ 2\ 2\ 2$?	800
$1\ 0\ \overset{0}{0}\ 0\ 0\ 0\ 0$	$2\ 2\ \overset{2}{0}\ 2\ 0\ 2\ 2$?	1040
$0\ 0\ \overset{0}{0}\ 0\ 0\ 0\ 1$	$2\ 2\ \overset{2}{0}\ 2\ 2\ 2\ 2$?	1520
$0\ 0\ \overset{0}{0}\ 0\ 0\ 0\ 0$	$2\ 2\ \overset{2}{2}\ 2\ 2\ 2\ 2$	2480

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