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ON THE STRONG ARNOL'D HYPOTHESIS AND THE CONNECTIVITY OF GRAPHS*

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Abstract. In the definition of the graph parameters $\mu(G)$ and $\nu(G)$, introduced by Colin de Verdière, and in the definition of the graph parameter $\xi(G)$, introduced by Barioli, Fallat, and Hogben, a transversality condition is used, called the Strong Arnol'd Hypothesis. In this paper, we define the Strong Arnol'd Hypothesis for linear subspaces $L \subseteq \mathbb{R}^n$ with respect to a graph $G = (V, E)$, with $V = \{1, 2, \ldots, n\}$. We give a necessary and sufficient condition for a linear subspace $L \subseteq \mathbb{R}^n$ with $\dim L \leq 2$ to satisfy the Strong Arnol'd Hypothesis with respect to a graph G, and we obtain a sufficient condition for a linear subspace $L \subseteq \mathbb{R}^n$ with $\dim L = 3$ to satisfy the Strong Arnol'd Hypothesis with respect to a graph G. We apply these results to show that if $G = (V, E)$ with $V = \{1, 2, \ldots, n\}$ is a path, 2-connected outerplanar, or 3-connected planar, then each real symmetric $n \times n$ matrix $M = [m_{i,j}]$ with $m_{i,j} < 0$ if $ij \in E$ and $m_{i,j} = 0$ if $i \neq j$ and $ij \notin E$ (and no restriction on the diagonal), having exactly one negative eigenvalue, satisfies the Strong Arnol'd Hypothesis.

Key words. Symmetric matrices, Nullity, Graphs, Transversality, Planar, Outerplanar, Graph minor.

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