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LEAST SQUARES (P,Q)-ORTHOGONAL SYMMETRIC SOLUTIONS OF THE MATRIX EQUATION AND ITS OPTIMAL APPROXIMATION*

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Abstract. In this paper, the relationship between the (P,Q)-orthogonal symmetric and symmetric matrices is derived. By applying the generalized singular value decomposition, the general expression of the least square (P,Q)-orthogonal symmetric solutions for the matrix equation $A^TXB = C$ is provided. Based on the projection theorem in inner space, and by using the canonical correlation decomposition, an analytical expression of the optimal approximate solution in the least squares (P,Q)-orthogonal symmetric solution set of the matrix equation $A^TXB = C$ to a given matrix is obtained. An explicit expression of the least square (P,Q)-orthogonal symmetric solution for the matrix equation $A^TXB = C$ with the minimum-norm is also derived. An algorithm for finding the optimal approximation solution is described and some numerical results are given.

Key words. Matrix equation, Least squares solution, (P,Q)-orthogonal symmetric matrix, Optimal approximate solution.

AMS subject classifications. 65F15, 65F20.

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