

THE STRUCTURE OF LINEAR PRESERVERS OF LEFT MATRIX MAJORIZATION ON \mathbb{R}^{P} *

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Abstract. For vectors $X, Y \in \mathbb{R}^n$, Y is said to be left matrix majorized by $X (Y \prec_{\ell} X)$ if for some row stochastic matrix R, Y = RX. A linear operator $T: \mathbb{R}^p \to \mathbb{R}^n$ is said to be a linear preserver of \prec_{ℓ} if $Y \prec_{\ell} X$ on \mathbb{R}^p implies that $TY \prec_{\ell} TX$ on \mathbb{R}^n . The linear operators $T: \mathbb{R}^p \to \mathbb{R}^n$ (n < p(p-1)) which preserve \prec_{ℓ} have been characterized. In this paper, linear operators $T: \mathbb{R}^p \to \mathbb{R}^n$ which preserve \prec_{ℓ} are characterized without any condition on n and p.

Key words. Row stochastic matrix, Doubly stochastic matrix, Matrix majorization, Weak matrix majorization, Left (right) multivariate majorization, Linear preserver.

AMS subject classifications. 15A04, 15A21, 15A51.

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