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SUBDOMINANT EIGENVALUES FOR STOCHASTIC MATRICES WITH GIVEN COLUMN SUMS*

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Abstract. For any stochastic matrix A of order n, denote its eigenvalues as $\lambda_1(A), \ldots, \lambda_n(A)$, ordered so that $1 = |\lambda_1(A)| \geq |\lambda_2(A)| \geq \ldots \geq |\lambda_n(A)|$. Let c^T be a row vector of order n whose entries are nonnegative numbers that sum to n. Define $\mathcal{S}(c)$, to be the set of $n \times n$ row-stochastic matrices with column sum vector c^T. In this paper the quantity $\overline{\lambda(c)} = \max\{|\lambda_2(A)||A \in \mathcal{S}(c)\}$ is considered. The vectors c^T such that $\overline{\lambda(c)} < 1$ are identified and in those cases, nontrivial upper bounds on $\overline{\lambda(c)}$ and weak ergodicity results for forward products are provided. The results are obtained via a mix of analytic and combinatorial techniques.

Key words. Stochastic matrix, Subdominant eigenvalue, Bipartite graph.

AMS subject classifications. 15A51, 15A18, 15A42, 60J10.

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