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MAPS ON POSITIVE OPERATORS PRESERVING LEBESGUE DECOMPOSITIONS*

LAJOS MOLNÁR†

Abstract. Let H be a complex Hilbert space. Denote by $B(H)^+$ the set of all positive bounded linear operators on H. A bijective map $\phi : B(H)^+ \to B(H)^+$ is said to preserve Lebesgue decompositions in both directions if for any quadruple A, B, C, D of positive operators, $B = C + D$ is an A-Lebesgue decomposition of B if and only if $\phi(B) = \phi(C) + \phi(D)$ is a $\phi(A)$-Lebesgue decomposition of $\phi(B)$. It is proved that every such transformation ϕ is of the form $\phi(A) = SAS^*$ $(A \in B(H)^+)$ for some invertible bounded linear or conjugate-linear operator S on H.

Key words. Positive operators, Lebesgue decomposition, Preservers.

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†Institute of Mathematics, University of Debrecen, H-4010 Debrecen, P.O. Box 12, Hungary (molnarl@math.klte.hu, http://www.math.klte.hu/~molnarl/). Supported by the Hungarian National Foundation for Scientific Research (OTKA), Grant No. NK68040, and by the Alexander von Humboldt Foundation, Germany.