

THE Q -MATRIX COMPLETION PROBLEM*

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Abstract. A real $n \times n$ matrix is a Q -matrix if for every $k = 1, 2, \dots, n$ the sum of all $k \times k$ principal minors is positive. A digraph D is said to have Q -completion if every partial Q -matrix specifying D can be completed to a Q -matrix. For the Q -completion problem, sufficient conditions for a digraph to have Q -completion are given, necessary conditions for a digraph to have Q -completion are provided, and those digraphs of order at most four that have Q -completion are characterized.

Key words. Partial matrix, Matrix completion, Q -matrix, Q -completion, Digraph.

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