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STABLE SUBNORMS ON FINITE-DIMENSIONAL POWER-ASSOCIATIVE ALGEBRAS*

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Abstract. Let $\mathcal{A}$ be a finite-dimensional power-associative algebra over a field $\mathbb{F}$, either $\mathbb{R}$ or $\mathbb{C}$, and let $\mathcal{S}$, a subset of $\mathcal{A}$, be closed under scalar multiplication. A real-valued function f on $\mathcal{S}$ is called a *subnorm* if $f(a) > 0$ for all $0 \neq a \in \mathcal{S}$, and $f(\alpha a) = |\alpha| f(a)$ for all $a \in \mathcal{S}$ and $\alpha \in \mathbb{F}$. If in addition, $\mathcal{S}$ is closed under raising to powers, then a subnorm f is said to be *stable* if there exists a positive constant σ so that

$$f(a^k) \leq \sigma f(a)^k \quad \text{for all } a \in \mathcal{S} \text{ and } k = 1, 2, 3, \dots \ .$$

The purpose of this paper is to provide an updated account of our study of stable subnorms on subsets of finite-dimensional power-associative algebras over $\mathbb{F}$. Our aim is to review and discuss some of the results in several previous papers, dealing with both continuous and discontinuous subnorms.

Key words. Finite-dimensional power-associative algebras, Norms, Subnorms, Submoduli, Stable subnorms, Minimal polynomial, Radius of an element in a finite-dimensional power-associative algebra.

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