Electronic Journal of Linear Algebra ISSN 1081-3810
A publication of the International Linear Algebra Society
Volume 17, pp. 304-315, June 2008

ELA
http://math.technion.ac.il/iic/ela

LINEAR PRESERVERS OF LEFT MATRIX MAJORIZATION*

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Abstract. For $X, Y \in M_{nm}(\mathbb{R})$ $(= M_{nm})$, we say that Y is left (resp. right) matrix majorized by X and write $Y \prec_\ell X$ (resp. $Y \prec_r X$) if $Y = RX$ (resp. $Y = XR$) for some row stochastic matrix R. A linear operator $T: M_{nm} \to M_{nm}$ is said to be a linear preserver of a given relation $\prec$ on M_{nm} if $Y \prec X$ implies that $TY \prec TX$. The linear preservers of $\prec_\ell$ or $\prec_r$ are fully characterized by A.M. Hasani and M. Radjabalipour. Here, we launch an attempt to extend their results to the case where the domain and the codomain of T are not necessarily identical. We begin by characterizing linear preservers $T : M_{p1} \to M_{n1}$ of $\prec_\ell$.

Key words. Row stochastic matrix, Doubly stochastic matrix, Matrix majorization, Weak matrix majorization, Left (right) multivariate majorization, Linear preserver.

AMS subject classifications. 15A04, 15A21, 15A51.

*Received by the editors February 27, 2008. Accepted for publication June 4, 2008. Handling Editor: Bit-Shun Tam.

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