# ON THE EXPONENT OF $R$-REGULAR PRIMITIVE MATRICES* 

M.I. BUENO ${ }^{\dagger}$ AND S. FURTADO ${ }^{\ddagger}$


#### Abstract

Let $P_{n r}$ be the set of $n$-by- $n r$-regular primitive $(0,1)$-matrices. In this paper, an explicit formula is found in terms of $n$ and $r$ for the minimum exponent achieved by matrices in $P_{n r}$. Moreover, matrices achieving that exponent are given in this paper. Gregory and Shen conjectured that $b_{n r}=\left\lfloor\frac{n}{r}\right\rfloor^{2}+1$ is an upper bound for the exponent of matrices in $P_{n r}$. Matrices achieving the exponent $b_{n r}$ are presented for the case when $n$ is not a multiple of $r$. In particular, it is shown that $b_{2 r+1, r}$ is the maximum exponent attained by matrices in $P_{2 r+1, r}$. When $n$ is a multiple of $r$, it is conjectured that the maximum exponent achieved by matrices in $P_{n r}$ is strictly smaller than $b_{n r}$. Matrices attaining the conjectured maximum exponent in that set are presented. It is shown that the conjecture is true when $n=2 r$.


Key words. $r$-Regular matrices, Primitive matrices, Exponent of primitive matrices

AMS subject classifications. 05C20, 05C50, 15A36.

[^0]
[^0]:    *Received by the editors May 01, 2007. Accepted for publication January 15, 2008. Handling Editor: Michael J. Tsatsomeros.
    ${ }^{\dagger}$ Department of Mathematics, University of California, Santa Barbara, CA USA (mbueno@math.ucsb.edu). Supported by Dirección General de Investigación (Ministerio de Ciencia y Tecnología) of Spain under grant MTM2006-06671.
    ${ }^{\ddagger}$ Faculdade de Economia do Porto, Rua Dr. Roberto Frias 4200-464 Porto, Portugal (sbf@fep.up.pt).

