

ON (CON)SIMILARITIES AND CONGRUENCES BETWEEN A AND A^*, A^T OR \overline{A} ¹

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Abstract. A unifying approach is presented between similarity, consimilarity, ^T congruence and *congruence of a matrix A to a symmetric, to a Hermitian or to a real matrix. Also studied are similarity consimilarity, ^T congruence and *congruence of a matrix A to A^*, A^T , and \overline{A} . Attempts are made to find special (con)similarities and congruences, as well as to find connections between these classes of maps. For example, it is shown that if $A\overline{A}$ is nonderogatory and nonsingular, then the consimilarities between A and A^T are precisely the Hermitian similarities between $A\overline{A}$ and $(A\overline{A})^*$. Also, if A is nonsingular, then the coninvolutory ^T congruences between A and A^* are in 1-to-1 correspondence with Hermitian similarities between $A(A^{-1})^T$ and $(A(A^{-1})^T)^*$.

Key words. Similarities, Consimilarities, $^T {\rm congruences},$ *congruences, Canonical forms, Involutions, Coninvolutions.

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