

SIGN PATTERNS THAT REQUIRE A POSITIVE OR NONNEGATIVE LEFT INVERSE*

IN-JAE KIM[†] AND BRYAN L. SHADER[‡]

Abstract. An m by n sign pattern \mathcal{A} is an m by n matrix with entries in $\{+, -, 0\}$. The sign pattern \mathcal{A} requires a positive (resp. nonnegative) left inverse provided each real matrix with sign pattern \mathcal{A} has a left inverse with all entries positive (resp. nonnegative). In this paper, necessary and sufficient conditions are given for a sign pattern to require a positive or nonnegative left inverse. It is also shown that for $n \geq 2$, there are no square sign patterns of order n that require a positive (left) inverse, and that an n by n sign pattern requiring a nonnegative (left) inverse is permutationally equivalent to an upper triangular sign pattern with positive main diagonal entries and nonpositive off-diagonal entries.

Key words. Nonnegative left inverse, Positive left inverse, Sign-consistent constrained system, Sign pattern.

AMS subject classifications. 15A06, 15A09, 15A48, 05C20.

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[†]Department of Mathematics and Statistics, Minnesota State University, Mankato, MN 56001, USA (in-jae.kim@mnsu.edu). The research of this author was partially supported by a Faculty Reassigned Time for Research and a Faculty Research Grant from the Minnesota State University, Mankato.

[‡]Department of Mathematics, University of Wyoming, Laramie, WY 82071, USA (bshader@uwyo.edu).