

CONSTRUCTING COPOSITIVE MATRICES FROM INTERIOR MATRICES*

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Abstract. Let A be an n by n symmetric matrix with real entries. Using the l_1 -norm for vectors and letting $S_1^+ = \{x \in \mathbb{R}^n \mid \|x\|_1 = 1, x \geq 0\}$, the matrix A is said to be *interior* if the quadratic form $x^T A x$ achieves its minimum on S_1^+ in the interior. Necessary and sufficient conditions are provided for a matrix to be interior. A copositive matrix is referred to as being *exceptional* if it is not the sum of a positive semidefinite matrix and a nonnegative matrix. A method is provided for constructing exceptional copositive matrices by completing a partial copositive matrix that has certain specified overlapping copositive interior principal submatrices.

Key words. Copositive matrix, Interior matrix, Quadratic form, Almost positive semidefinite, Exceptional copositive matrix, Extreme copositive matrix.

AMS subject classifications. 15A18, 15A48, 15A57, 15A63.

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